Krasovsky Institute of Mathematics and Mechanics
Sobolev Institute of Mathematics
Melentiev Energy Systems Institute
Higher School of Economics, Nizhny Novgorod
Ural Federal University
Novosibirsk State University
Ural Mathematical Center
Ural Branch of the Russian Academy of Science
Mathematical Center in Academgorodok

XXII International Conference

Mathematical Optimization Theory and Operations Research

(MOTOR 2023)

Ekaterinburg, Russia

July 2–8, 2023

http://motor2023.uran.ru

Abstracts

Ekaterinburg · Russia · 2023

This conference is devoted to 90^{th} anniversary of academician I.I. Eremin

XXII International Conference "Mathematical Optimization Theory and Operations Research" (MOTOR 2023). Abstracts / M. Khachay, Y. Kochetov, A. Eremeev, O. Khamisov, V. Mazalov, and P. Pardalos (Eds.).— Ekaterinburg, Russia: Publisher "UMC UrFU", 2023. — 125 p.

This volume contains abstracts submited to the 22^{nd} International Conference on Mathematical Optimization Theory and Operations Research (MOTOR 2023) held in Ekaterinburg, Russia on Jul. 2-8, 2023.

ISBN

CONTENTS

1. **Plenary Lectures** 4
2. **Industrial Talks** 12
3. **Mathematical Programming and Applications** . . . 14
4. **Discrete and Combinatorial Optimization** 27
5. **Stochastic Optimization** 43
6. **Scheduling** 49
7. **Operations Research** 55
8. **Game Theory** 79
9. **Optimal Control and Mathematical Economics** . . 91
10. **Optimization in Machine Learning and Artificial Intelligence** 107

Author Index 118

1. Plenary Lectures

Prof. Kamil Aida-Zade and Samir Guliev

FEEDBACK CONTROL ON THE CLASS OF ZONAL CONTROL ACTIONS

Institute of Control Systems, Baku, Azerbaijan
kamil_aydazade@rambler.ru

We propose an approach to feedback control for both nonlinear objects with lumped parameters and point sources of objects with distributed parameters. The approach lies in the fact that the entire set of possible phase states of an object is divided into a finite number of zones (subsets), and the synthesized controls are determined not by the measured state values themselves, but by the zonal values of the zone parameters to which belong the current measured object states. We have obtained necessary optimality conditions for the zonal values of the feedback parameters, carried out computational (computer) experiments on some test problems of feedback control both for objects with lumped parameters and for distributed parameters.

keywords: feedback control, optimality conditions.

Prof. Mario R. Guarracino

SEMI-SUPERVISED LEARNING WITH DEPTH FUNCTIONS

Higher School of Economics, Russia
mario.guarracino@gmail.com

Depth functions have been exploited in supervised learning since years. Given that the depth of a point is somehow a distribution-free measure of its distance from the center of a distribution, their use in supervised learning arose naturally and it has seen a certain degree of success. Particularly, DD-classifers and their extensions have been extensively studied and applied in many applied fields and statistical settings. What has not been investigated so far is their use within a semi-supervised learning framework. That is, in case some labeled data are available along with some unlabeled data within the same training set. A case which arises in many applications and where it has been proved that combining information from labeled and unlabeled data can improve the overall performance of a classifier. For this reason, this work aims at introducing semi-supervised learning techniques in association with DD-classifiers and at investigating to what extent such technique is able to improve DD-classifier performances. Performances will be evaluated by means of an extensive simulation study and illustrated on some real data sets.

keywords: semi-supervised learning techniques, depth functions, performance evaluation.

Prof. Milojica Jaćimović

STRONG CONVERGENCE OF EXTRAGRADIENT-LIKE METHODS FOR SOLVING QUASI-VARTIATIONAL INEQUALITIES

University of Montenegro, Podrogica, Montenegro
milojica@jacimovic.me

The goal of this talk is to study convergence of approximation methods for quasi-variational inequalities with the moving set. First, we propose the extragradient dynamical system and under strong monotonicity we show strong convergence with exponential rate of generated trajectory to the unique solution of quasi-variational inequality. Further, the explicit time discretization of this dynamical system leads to an extragradient algorithm with relaxation parameters. We prove the convergence of the generated iterative sequence to the unique solution of the quasi-variational inequality and derive the linear convergence rate under strong monotonicity.

keywords: quasi-variational inequalities, extragradient dynamical system, convergence rate.

Prof. Pinyan Lu

ALGORITHMS FOR SOLVERS: IDEAS FROM CS AND OR

Shanghai University of Finance and Economics, China
lu.pinyan@mail.shufe.edu.cn

The MIP/LP solvers are primarily developed by the operations research community while SAT/SMT solvers are primarily developed by the computer science community. However, these problems are closely related with each other. In recent years, there have been many attempts to combine the algorithmic techniques of both sides to develop better solvers. In this talk, I will discuss about these and our attempts and try to provide a unified perspective and framework.

keywords: MIP and SAT solvers, efficient algorithms, general framework.

Prof. Panos Pardalos

ARTIFICIAL INTELLIGENCE, SMART ENERGY SYSTEMS, AND SUSTAINABILITY

University of Florida, USA
p.m.pardalos@gmail.com

Distribution systems face significant changes due to the growing number of distributed and variable energy generation resources and the smart grid implementation. The traditional design paradigm can no longer meet the need for greater resilience, power quality, and customer participation. On the other hand, smart grid implementation brings a large amount of data that can be

used to better plan a distribution system. Growing energy demand and limited investment capital make distribution system planners look to these advances in smart grid technology to identify new approaches to achieve load reliability. When planning a distribution system, the main goal is to meet the most economically and reliably timed demand growth. The planning methodology must ensure that every opportunity for savings or power quality improvement is exploited. This is not a straightforward task, even in traditional systems, since the distribution networks are usually large in extension, with a large amount of data to be analyzed. In addition, new regulations from authorities and the modernization of power systems highlight the importance of a constant update and improvement of methodologies and planning techniques. The ongoing changes bring enormous opportunities and challenges to traditional and new players requiring huge planning and operation methods changes. With more and innovative players entering the sector, artificial intelligence-based approaches can be the key to dealing with the new challenges and ensuring the systems and the respective players' sustainability, both in economic and environmental terms. The drive to make utilities more efficient through AI, machine learning, and data science has resulted in major benefits for every actor in the energy sector, including generators, distributors, the environment, taxpayers, and consumers.

keywords: sustainability, smart energy and distribution systems, artificial intelligence.

Prof. Eugene Semenkin

HYBRID EVOLUTIONARY OPTIMIZATION:
HOW SELF-ADAPTED ALGORITHMS CAN AUTOMATICALLY GENERATE APPLIED AI TOOLS

Reshetnev Siberian State University of Science and Technology, Russia
eugenesemenkin@yandex.ru

When designing AI tools or machine-learning models, one must make multiple choices: which approach should be used, which structure of model is more appropriate for the problem in hand and which settings and parameters must be applied. All of these choices are mathematically reduced to some kind of optimization problem. In fact, one has to solve multi-criteria multi-dimensional multi-scale "black box" optimization problems with algorithmically-given objectives and/or constraints. Bio-inspired algorithms, e.g. evolutionary algorithms (EAs), could be used for solving the described problems. However, the effectiveness and efficiency of EAs depends essentially on the choice of their settings and the tuning of their parameters, which is a separate and very complicated decision-making problem. An EA with appropriate settings and parameters can be very effective, but in the opposite case, this algorithm can fail. Moreover,

such a useful property of EAs as their universality (their independence from the properties of the problem), which allows them to be used in solving the widest class of optimization problems, means that it is impossible to use properties of problems convenient for optimization (such as convexity, monotony and unimodality) in cases where such properties exist in the problem being solved. The approach that will be discussed in the lecture allows us to simplify the use of EAs and other bio-inspired algorithms by means of the self-configuration, coevolution and hybridization of EAs with problem-specific algorithms. Details of the approach will be described, and examples of approach deployment in the automated design of AI-tools will be given.

keywords: artificial intelligence tools, evolutionary algorithms, black-box optimization.

Prof. Yaroslav D. Sergeyev

NUMERICAL INFINITIES AND INFINITESIMALS IN OPTIMIZATION

University of Calabria, Italy
yaro@dimes.unical.it

In this talk, a recent computational methodology is described (see [1,2]). It has been introduced with the intention to allow one to work with infinities and infinitesimals numerically in a unique computational framework. It is based on the principle 'The part is less than the whole' applied to all quantities (finite, infinite, and infinitesimal) and to all sets and processes (finite and infinite). The methodology uses as a computational device the Infinity Computer (a new kind of supercomputer patented in several countries) working numerically with infinite and infinitesimal numbers that can be written in a positional system with an infinite radix. On a number of examples (numerical differentiation, divergent series, ordinary differential equations, fractals, set theory, etc.) it is shown that the new approach can be useful from both theoretical and computational points of view. The main attention is dedicated to applications in optimization (local, global, and multi-objective)(see [1,2-7]). The accuracy of the obtained results is continuously compared with results obtained by traditional tools used to work with mathematical objects involving infinity. The Infinity Calculator working with infinities and infinitesimals numerically is shown during the lecture. For more information see the dedicated web page http://www.theinfinitycomputer.com and this survey: The web page developed at the University of East Anglia, UK is dedicated to teaching the methodology: https://www.numericalinfinities.com/.

keywords: numerical infinities and infinitesimals, optimization techniques, infinity computer.

References

1. Sergeyev Ya.D. and De Leone R., eds, Numerical Infinities and Infinitesimals in Optimization. Springer, Cham, 2022.

2. Sergeyev Ya.D. Numerical infinities and infinitesimals: Methodology, applications, and repercussions on two Hilbert problems. EMS Surveys in Math. Sci., 2017, 4(2), 219–320.
3. De Leone R., Egidi N., Fatone L. The use of grossone in elastic net regularization and sparse support vector machines. Soft Comp., 24, 17669-17677, 2020.
4. Astorino A., Fuduli A. Spherical separation with infinitely far center. Soft Comp., 24, 17751-17759, 2020.
5. De Leone R., Fasano G., Roma M., Sergeyev Y.D. Iterative grossone-based computation of negative curvature directions in large-scale optimization. JOTA, 186(2), 554-589, 2020.
6. De Leone R., Fasano G., Sergeyev Ya.D. Planar methods and grossone for the Conjugate Gradient breakdown in nonlinear programming. Comp. Opt. and Appl., 71(1), 73-93, 2018.
7. Cococcioni M., Pappalardo M., Sergeyev Ya.D. Lexicographic multiobjective linear programming using grossone methodology: Theory and algorithm. Appl. Math. and Comp., 318, 298–311, 2018.

Prof. Alexander Shananin and Dr. Natalia Obrosova

General equilibrium models in production networks with substitution of inputs

Lomonosov Moscow State University, Russia
alexshan@yandex.ru

The developed countries made a transition from intensive to extensive growth in 1980s, that led to goods and services range extension. Related changes in the technologies during the economic globalization caused supply chains sophistication and complex production network development in local and global economies. The problem of sustainable development of regional economies while supply chains was localized raised due to latest pandemic and sanction economic shocks. Proper tools are needed to describe modern production networks taking into account the substitution of inputs. Traditional interindustry balance method based on the Leontief linear model and nonnegative matrices theory should not be used due to its base assumption of the constancy of direct requirement coefficients in a supply network. Modern methods demand new mathematical tools that reflect substitution of inputs in complex production networks. Tools presented by authors developed on the results of the analysis of the resource allocation problem with positively homogeneous neoclassical production functions and the dual problem which solutions are the price indexes of intermediate inputs. The way of problem studying is based on the construction of the Young dual transform for equilibrium price indexes. It is proved that in the case of CES technologies with constant elasticity of substitution (Cobb-Douglas in particular) resource allocation problem has an explicit solution. In this case the comparative statics method based on the official national accounts statistics is developed by authors. Method allows in a middle-term to forecast intersectoral links under given scenarios of the internal or external shocks taking into account the substitution of inputs. The point of the method is solving the inverse problem of nonlinear balance identification and further verification of the model on the base of the official input-output tables statistics. By that the elasticities of substitution of intermediate inputs are the verification parameters of the model. The method was successfully tested on

the input-output tables statistics of Russia and Kazakhstan. Applications to the analysis of the intersectoral links of several countries with different levels of market relations maturity demonstrated that in the case of decentralized economy the nonlinear balance with Cobb-Douglas production functions gives more precise forecast than traditional linear Leontief model approach.

keywords: production networks, equilibrium models.

Prof. Predrag S. Stanimirović

Optimization methods in gradient and zeroing neural networks

University of Niš, Serbia

predrag.stanimirovic@pmf.edu.rs

The topic of our lecture is a class of recurrent neural networks (RNN) dedicated to finding zeros of equations or minimizing nonlinear functions. Optimization RNN models are divided into two global classes: Gradient Neural Networks (GNN) and Zhang Neural Networks (ZNN). GNN models are aimed at solving time-invariant problems, while ZNN models are able to solve time-varying problems. The design of GNN and ZNN models arises from the choice of an appropriate error function. ZNN dynamics is a certain dynamical system whose states evolve over a state space continuously based on the time derivative of the error function. Some new error functions resulting from nonlinear gradient-descent and quasi-Newton optimization methods are presented. A modification of ZNN dynamical evolution based on higher-order hyperpower iterative methods is described. We discuss the problems of non-differentiability and division by zero (DBZ) which appear relatively frequently in time-varying dynamical systems. The GNN design is defined as a movement along the negative gradient of the Frobenius norm of the error function inside the time interval. Modifications of the GNN flow based on gradient-descent and conjugate-gradient optimization methods are considered. In general, dynamical systems are defined as continuous-time analogies of known nonlinear optimization algorithms, such as the class of gradient-descent algorithms or various quasi-Newton method for solving nonlinear optimization problems. The convergence of various modified dynamical systems aimed to solving time-varying matrix equations are investigated.

keywords: recurrent, gradient and Zhang neural networks, gradient-descent and quasi-Newton methods.

Prof. Vladimir V. Vasin

ITERATIVE PROCESSES OF FEJÉR TYPE FOR QUADRATIC OPTIMIZATION PROBLEM

Krasovsky Institute of Mathematics and Mechanics, Ekaterinburg, Russia
vasin@imm.uran.ru

This lecture presents a short overview of iterative solution methods of Fejér type for the well-known quadratic constrained optimization problem, which were introduced and widely studied by acad. I.I. Eremin — the founder of the Ural mathematical programming school. Along with common quadratic programs, we consider several variants of the basic problem, which have numerous applications. We point attention to some special settings of the problem in question including metric projections, linear programming, etc., which are of separate interest.

Fejér-type methods adopt interest of specialists on numerical optimization, since, along with convergence, one can prove conditions of their stability to small perturbations of input data. Thus, these methods induce self-regularizing algorithms, unlike some well-known primal numerical optimization techniques.

keywords: Fejér iterative processes, quadratic optimization, stability.

2. Industrial Talks

Dr. Mikhail Krasilnikov

The principles of development and implementation of decision making software solutions in LTL-company

BIA Technologies, Russia
Mihail.Krasilnikov@bia-tech.ru

This talk will shine a spotlight on the challenges software developer faces when applying Operations Research methods in the development and implementation of software solutions. Typically, large transportation companies consist of a number of branches, including production (those who own vehicles), sales (those who sell company services and must know what prices will be set for each individual type of vehicle and each destination), and marketing (those who manage demand by setting up special offers and discounts). The process of operations planning for different branches is interconnected. As a result, the company needs a planning system on both the department and global levels. Planning is required over different time horizons, interdepartmental planning often has a greater planning horizon. The development of these systems has a multitude of complex requirements. This talk will touch upon the implementation of a few systems, one of which—last-mile planning is a multi-depot vehicle routing problem with time windows. Another problem that is going to be discussed is assigning a vehicle to a transportation request, which in an essence is the assignment problem. Implementation steps, infrastructure, and algorithm requirements will also be explored. The challenges of implementation are important to mention since the automated planning process is different from what end-users are used to, so a level of trust has to be achieved. Finally, we will talk about possible ways to further the progress of implementation of these technologies into a real-world context, for example, by means of open source projects.

3. Mathematical Programming and Applications

Savelii Chezhegov[1], **Alexander Rogozin**[1] **and Alexander Gasnikov**[1,2,3]

On decentralized nonsmooth optimization

[1] Moscow Institute of Physics and Technology, Russia
[2] Institute for Information Transportation Problems, Russia
[3] Caucasus Mathematical Center of Adygh State University, Russia
gasnikov.av@mipt.ru

In decentralized optimization, several nodes connected by a network collaboratively minimize some objective function. For minimization of Lipschitz functions, lower bounds are known along with optimal algorithms. We study a specific class of problems: linear models with nonsmooth loss functions. Our algorithm combines regularization and dual reformulation to get an effective optimization method with complexity better than the lower bounds.

keywords: convex optimization, distributed optimization.

Nikita Fedin[1] **and Eduard Gorbunov**[2]

Byzantine-robust loopless stochastic variance-reduced gradient

[1] Moscow Institute of Physics and Technology, Russia
[2] Mohamed bin Zayed University of Artificial Intelligence,UAE
fedin.ng@phystech.edu, eduard.gorbunov@mbzuai.ac.ae

Distributed optimization with open collaboration is a popular field since it provides an opportunity for small groups / companies / universities, and individuals to jointly solve huge-scale problems. However, standard optimization algorithms are fragile in such settings due to the possible presence of so-called Byzantine workers – participants that can send (intentionally or not) incorrect information instead of the one prescribed by the protocol (e.g., send anti-gradient instead of stochastic gradients). Thus, the problem of designing distributed methods with provable robustness to Byzantine workers has been receiving a lot of attention recently. In particular, several works consider a very promising way to achieve Byzantine tolerance via exploiting variance reduction and robust aggregation. The existing approaches use SAGA- and SARAH-type variance reduced estimators, while another popular estimator - SVRG - is not studied in the context of Byzantine-robustness. In this work, we close this gap in the literature and propose a new method - Byzantine- Robust Loopless Stochastic Variance Reduced Gradient (BR-LSVRG). We derive nonasymptotic convergence guarantees for the new method in the strongly convex case and compare its performance with existing approaches in numerical experiments.

keywords: distributed optimization, Byzantine-robustness, variance reduction, stochastic optimization.

Tatiana Gruzdeva[1] **and Anton Ushakov**[1]

SEMI-SUPERVISED K-MEANS CLUSTERING VIA DC PROGRAMMING APPROACH

[1] Matrosov Institute for System Dynamics and Control Theory, Russia
gruzdeva@icc.ru, aushakov@icc.ru

Though clustering is related to unsupervised machine learning and does not require any prior information on data items, in many real-life settings, there may be some expert knowledge on data labels or the properties of clusters known in advance. Obviously, such knowledge may be used to guide clustering process and improve the quality of found partitions. The clustering problems that involve some additional information on tags are called semi-supervised or constrained clustering problems. One distinguishes instance-level and cluster-level constraints usually formalized as the so-called must-link and cannot-link constraints or minimum/maximum cluster size. In this paper, we consider the constrained minimum sum-of-squares (k-means) clustering (MSSC) problem that incorporates both instance- and cluster-level constraints. As far as we know, such a semi-supervised MSSC problem has not been considered in the literature yet. We formulate this clustering problem and some of its particular cases as DC (difference of convex) optimization problems. Then, we develop a solution approach based on a special local search method. We carry out computational experiments on test problem instances demonstrating the efficiency of the proposed solution approach.

keywords: semi-supervised clustering, k-means, minimum-sum-of-squares clustering, DC programming, local search, must-link, cannot-link, constrained clustering.

Olga Pinyagina

AN ALGORITHM FOR DECENTRALIZED MULTI-AGENT FEASIBILITY PROBLEMS

[1] Kazan Federal University, Kazan, Russia
Olga.Piniaguina@kpfu.ru

We consider the feasibility problem in a multi-agent decentralized form, where each agent has the personal information of the feasible subset, which is unknown to other agents. The common feasible set is composed of the agents' feasible subsets. For solving this problem, we reformulate it in the form of a variational inequality and propose an algorithm based on the projection method. Preliminary test calculations confirm the efficiency of the proposed approach.

keywords: feasibility problem, decentralized system, projection method, variational inequality.

Oleg Savchuk[1,2], **Fedor Stonyakin**[1,2], **Mohammad Alkousa**[1,3], **Rida Zabirova**[1], **Alexander Titov**[1] **and Alexander Gasnikov**[1,3,4,5]

ONLINE OPTIMIZATION PROBLEMS WITH FUNCTIONAL CONSTRAINTS UNDER RELATIVE LIPSCHITZ CONTINUITY AND RELATIVE STRONG CONVEXITY CONDITIONS

[1] Moscow Institute of Physics and Technology, Russia
[2] Vernadsky Crimean Federal University, Russia
[3] HSE University, Russia
[4] Kharkevich Institute for Information Transmission Problems, Russia
[5] Caucasus Mathematical Center, Adyghe State University, Russia
oleg.savchuk19@mail.ru, fedyor@mail.ru
mohammad.alkousa@phystech.edu, a.a.tytov@gmail.com
zabirova.rr@phystech.edu, gasnikov.av@phystech.edu

In this work, we consider the problem of strongly convex online optimization with convex inequality constraints. A scheme with switching over productive and non-productive steps is proposed for these problems. The convergence rate of the proposed scheme is proven for the class of relatively Lipschitz-continuous and strongly convex minimization problems. Moreover, we study the extensions of the Mirror Descent algorithms that eliminate the need for a priori knowledge of the lower bound on the (relative) strong convexity parameters of the observed functions. Some numerical experiments were conducted to demonstrate the effectiveness of one of the proposed algorithms with a comparison with another adaptive algorithm for convex online optimization problems.

keywords: online optimization, strongly convex programming problem, relatively Lipschitz-continuous function, relatively strongly convex function, mirror descent, regularization.

Rashid Yarullin[1] **and Igor Zabotin**[1]

A CUTTING METHOD WITH SUCCESSIVE USE OF CONSTRAINT FUNCTIONS IN CONSTRUCTING APPROXIMATING SETS

[1] Kazan Federal University, Russia
yarullinrs@gmail.com, iyazabotin@mail.ru

We propose a cutting method with an approximation of the constraint region for solving a conditional minimization problem. The developed method is characterized by the fact that when constructing approximating sets, there is a consistent use of constraint functions. This approach of considering constraints is implemented in such a way that only one constraint is used at the initial stage, and the number of involved constraints is increased as iteration points reach the admissible set. As a result, the proposed method uses a less amount of computational operations for constructing approximating sets, which is favorably distinguished from the known cutting methods. We discuss various

implementations of the sequential use of constraint functions. The convergence of the developed cutting method is proved, and an estimation of the solution accuracy is obtained for the proposed method.

keywords: nondifferentiable optimization, high-dimensional optimization problems, convex function, subgradient, cutting method, cutting plane, approximating set, polyhedral approximation, consistent use of constraints, estimation of the solution accuracy.

This paper has been supported by the Kazan Federal University Strategic Academic Leadership Program ("PRIORITY-2030").

Igor Zabotin[1], **Oksana Shulgina**[1] **and Rashid Yarullin**[1]

IMPLEMENTING ONE VARIANT OF THE SUCCESSIVE CONCESSIONS METHOD FOR THE MULTI-OBJECTIVE OPTIMIZATION PROBLEM

[1] Kazan Federal University, Kazan, Russia

iyazabotin@mail.ru, onshul@mail.ru, yarullinrs@gmail.com

We propose one variant of the successive concessions method for solving a multi-objective optimization problem, which differs from the mentioned famous method by the approach of determining concessions. The concessions are set in the proposed method in such a way that the solutions found for the problems of the current and previous stages differ from each other not by the value of the objective functions, but by some distance no more than a given value. We describe the implementation of the method in cases when the problem of each stage is a convex pro- gramming problem. This implementation uses the algorithm proposed and proved in this paper, which belongs to the class of cutting methods with approximating the feasible set by polyhedral sets.

keywords: multi-objective optimization, successive concessions method, convex programming, approximating set, polyhedral approximation, cutting plane Approximations sequence, convergence, generalized-support vectors.

Vladimir Zubov[1]

ON THE UNIQUENESS OF IDENTIFICATION THE THERMAL CONDUCTIVITY AND HEAT CAPACITY OF SUBSTANCE

[1] Federal Research Center "Computer Science and Control", Russia

vladimir.zubov@mail.ru

The paper describes and investigates an algorithm for simultaneous identification the temperature-dependent volumetric heat capacity and thermal conductivity of a substance that is based on the results of experimental observation of the temperature field dynamics in the object. The algorithm is based on the first boundary value problem for a one-dimensional non-stationary heat equation. The considered inverse coefficient problem is reduced to a variational problem, that is solved by gradient methods. The gradient of the cost

functional is calculated with the help of the Fast Automatic Differentiation technique. The results of numerical solution to the considered inverse problem are presented. It is found that some inverse problems have a single solution, the solution of other problems is not the only one. The conditions of uniqueness of the solution to the inverse problem are formulated and justified. A class of temperature fields from the reachable sets is specified, for which the solution to the identification problem under consideration will be non-unique.

keywords: thermal conductivity, inverse coefficient problems, heat equation, gradient.

Seydamet Ablaev[1,2], **Fedor Stonyakin**[1,2], **Mohammad Alkousa**[1,3] **and Inna Baran**[2]

ADAPTIVE SUBGRADIENT METHODS FOR QUASICONVEX OPTIMIZATION PROBLEMS: FUNCTIONAL CONSTRAINTS AND SHARP MINIMA

[1] Moscow Institute of Physics and Technology, Russia
[2] Vernadsky Crimean Federal University, Russia
[3] HSE University, Russia
seydamet.ablaev@yandex.ru, fedyor@mail.ru
mohammad.alkousa@phystech.edu, matemain@mail.ru

The paper is devoted to the problem of minimizing a function with inequality-type constraints. It is known that if the sharp minimum condition is satisfied, then the linear convergence rate of the subgradient method can be guaranteed on a class of convex Lipschitz problems (including the non-smooth case). We investigate the possibility of generalizing this result to the class of quasiconvex Lipschitz-continuous objective functions with Lipschitz functional constraints. To do this, we select the class of problems with some "constrained" modification of sharp minimum and use the approach for investigation of subgradient methods proposed by Yu. Nesterov. for non-smooth problems without sharp minima. We study an adaptive subgradient method for quasiconvex problems with Lipschitz-continuous constraints and the considered modification of a sharp minimum.

keywords: Lipschitz-continuous function, quasiconvex function, sharp minimum, subgradient method, non-smooth problems.

Grigory Aivazian[1], **Mohammad Alkousa**[1,3] **and Fedor Stonyakin** [1,2]

SOME ADAPTIVE VARIANT OF THE FRANK-WOLFE ALGORITHM FOR CONVEX OPTIMIZATION PROBLEMS

[1] Moscow Institute of Physics and Technology,Russia
[2] Vernadsky Crimean Federal University, Russia
[3] HSE University, Russia
A1@phystech.edu, mohammad.alkousa@phystech.edu, fedyor@mail.ru

In this work, we researched a variant of the Frank-Wolfe method for convex

optimization problems with adaptive selection of the step parameter corresponding to information about the smoothness of the objective function (the Lipschitz constant of the gradient). Theoretical estimates of the quality of the solution provided by the method are obtained in terms of adaptively selected parameters L_k. An important feature of the obtained result is the elaboration of a situation in which it is possible to guarantee, after the completion of the iteration, a reduction of the discrepancy in the function by at least 2 times. At the same time, using of adaptively selected parameters in theoretical estimates makes it possible to apply the method for both smooth and nonsmooth problems, provided that the exit criterion from the iteration is met. For smooth problems, this can be proved, and the theoretical estimates of the method are guaranteed to be optimal up to multiplication by a constant factor. The results of computational experiments which show the possibility of applying the findings to some non-smooth problems are discussed.

keywords: convex optimization, Frank-Wolfe method, adaptivity, smooth problems, non-smooth problems.

Maria Barkova[1] **and Alexander Strekalovsky**[1]

On solving DC optimization problem with inequality constraints

[1] Matrosov Institute for System Dynamics and Control Theory, Russia

We address a nonconvex optimization problem with inequality constraints, where all functions can be represented as DC functions. We reduce the original problem to a penalized one without nonconvex constraints, which turns out to be the DC minimization problem. Therefore, it can be solved using the Global Search Theory which consists of two basic stages: a special local search method and procedures of escaping from a critical point based on Global Optimality Conditions. The local search method (LSM) employs a consecutive solution of linearized problems combined with a change of penalty parameter. Computational testing of developed methods has been carried out on specially generated test problems with known global solutions. The numerical results demonstrate the effectiveness of the proposed approach in comparison with modern software packages.

keywords: DC functions, nonconvex problem, exact penalty, local search method, global search, global optimality conditions.

Vladimir Erokhin[1]

Stable solution of a dual pair of approximate and improper linear programming problems with interval uncertainty of coefficients

[1] Mozhaisky Military Space Academy, Russia

erohin_v_i@mail.ru

Consider a dual pair of linear programming (LP) problems with interval coefficients. It is known that there exists a hypothetical exact proper pair of dual LP problems that satisfy the indicated intervals. Based on the classical theory of duality of LP problems, the theory of systems of linear interval inequalities, methods of regularization and matrix correction of unstable and improper LP problems, developed in the works of I.I. Eremin, A.A. Vatolin, A.N. Tikhonov, V.A. Morozov, G.M. Agayan, F.P. Vasilyev, A.Yu. Ivanitsky, V.A. Gorelik, a number of constructive results can be obtained. These results include necessary and sufficient conditions for the existence and form of normal solutions of a dual pair of interval LP problems and their convergence to normal solutions of a hypothetical exact dual pair of LP problems. The necessary theoretical calculations and illustrative numerical examples are given.

keywords: linear programming, interval uncertainty, regularization, matrix correction.

Battur Gompil[1], **Batbileg Sukhee**[1] **and Rentsen Enkhbat**[2]

GLOBAL OPTIMIZATION DC APPROACH TO QUADRATIC NONCONVEX GENERALIZED NASH EQUILIBRIUM PROBLEMS

[1] National Universoty of Mongolia, Mongolia

[2] Institute of Mathematical and Digital Technology, Mongolia

batbileg@seas.num.edu.mn, renkhbat46@yahoo.com

In this paper, we consider N players nonconvex quadratic generalized Nash equilibrium problems (GNEP) with jointly convex constraints.We show that the problem can be equivalently reduced to the nonconvex unconstrained optimization problem based on a regularized Nikaido-Isoda function and a gap function. The last problem is also nonconvex, so we reduce it to DC (difference of convex functions) optimization problem and then apply local and global methods and algorithms developed by A.S.Strekalovsky. Numerical results are presented.

keywords: nonconvex games, Nikaido-Isoda function, gap functions, DC optimization, local and global search methods.

Oleg Khamisov[1] **and Mikhail Posypkin**[1]

GENERALIZED CONVEXITY AND GLOBAL OPTIMIZATION

[1] Melentiev Energy Systems Institute, Russia

[2] Federal Research Center for Computer Science and Control, Russia

globopt@mail.ru, mposypkin@frccsc.ru

We use different concepts of the generalized convextity in oder to improve efficiency of different global optimization techniques. The main concept of

the generalized convextity is so called semi-strict quasiconvexity and under global optimization techniques Lipschitz optimizarion and branch and bound approach are understood. Theoretical justification as well as testing results are provided.

keywords: generalized convextity, global optimization techniques, semi-strict quasiconvexity, Lipschitz optimizarion, branch and bound.

Oleg Khamisov[1]

GLOBAL OPTIMIZATION IN CONVEX PARAMETRIC PROGRAMMING

[1] Melentiev Energy Systems Institute, Russia

globopt@mail.ru

We consider a convex optimization problem with vector parameter in the objective and in the constraints. The aim of consideration consists in finding upper and lower bounds on parametric value function. The suggested metodology is based on convex and concave support functions. The corresponding methods are described and theoretically justified, testing results are given.

keywords: convex optimization, upper and lower bounds, parametric value function, convex and concave support functions.

Vladimir Krutikov[1], **Predrag Stanimirovic**[2], **Lev Kazakovtsev**[4,5] **and Elena Tovbis**[4]

A FAMILY OF MULTI-STEP RELAXATION SUBGRADIENT MINIMIZATION METHODS FOR NON-SMOOTH FUNCTIONS

[1] Kemerovo State University, Russia

[2] University of Niš, Serbia

[3] Reshetnev Siberian State University of Science and Technology, Russia

[4] Siberian Federal University, Russia

krutikovvn@rambler.ru, levk@bk.ru

For solving multidimensional unconstrained optimization problems with a nonsmooth objective function, we present a relaxation subgradient methods (RSM) with a built-in algorithm for finding the descent direction that forms an acute angle with all subgradients in the neighborhood of the current solu-tion. Minimizing the function along the opposite direction (with a minus sign) enables the algorithm to go beyond the neighborhood of the current mini-mum. The family of algorithms for finding the descent direction is based on solving systems of inequalities. We propose a family of iterative algorithms $A(k)$ for solving systems of inequalities which differ in their parameter $A(k)$. For $k = 0$, our algorithm implements a scheme based on the Kacz-marz algorithm. The developed methods are substantiated theoretically; esti-mates of their convergence rate are

obtained. On this basis, a family of re-laxation subgradient minimization algorithms is formulated and justified, ap-plicable to solving nonconvex problems as well. According to the properties of convergence on quadratic functions of high dimension, with large spreads of eigenvalues, the developed algorithms are equivalent to the conjugate gra-dient method. The new methods enable us to solve non-smooth non-convex large-scale minimization problems with a high degree of elongation of level surfaces. Finite convergence of algorithms on separable bounded sets is proved. Examples of solving convex and non-convex smooth and non-smooth problems of large dimensions are given.

keywords: unconstrained optimization, non-smooth objective function, relaxation subgradient methods (RSM), nonconvex problems.

Vladimir Krutikov[1], **Predrag Stanimirovic**[2], **Lev Kazakovtsev**[4,5] and **Elena Tovbis**[4]

ALGORITHM FOR CONVEX CONSTRAINED OPTIMIZATION WITH A QUADRATIC TRANSFORMATION OF DUAL VARIABLES

[1] Kemerovo State University, Russia
[2] University of Niš, Serbia
[3] Reshetnev Siberian State University of Science and Technology, Russia
[4] Siberian Federal University, Russia
krutikovvn@rambler.ru, levk@bk.ru

In our algorithm, we construct a dual constrained optimization problem on the positivity of variables, for a convex programming problem with a strongly convex objective function. Afterwards, we move to an unconstrained optimization ptoblem via transformations of the components of dual variables. We show that in the case of using the absolute values of dual variables components, the degree of the function degeneracy increases as it approaches the optimal point. The gradient ambiguity in the regions of the sign change leads to the necessity of using special methods such as relaxation subgradient methods for non-smooth non-convex optimization problems with a high degree of elongation of level surfaces. We demonstrate that the quadratic transformation of the variable components enables us to obtain a dual function with a Lipschitz gradient and use much simpler gradient methods to find the extremum. The efficiency of the new algorithm is confirmed by a computational experiment. The quadratic transformation compared to the absolute value transformation rnables us to obtain a solution to the problem by both relaxation subgradient methods and smooth function minimization methods (conjugate gradient method and quasi-Newtonian method) quicker with higher accuracy.

keywords: convex programming, nonlinear programming, dual problem, subgradient methods, gradient optimization methods.

Ilya Kuruzov[1] **and Fedor Stonyakin**[1]

ON SOME VERSIONS OF SUBSPACE OPTIMIZATION METHODS WITH INEXACT GRADIENT INFORMATION

[1] Moscow Institute of Physics and Technology, Russia
fedyor@mail.ru

Inexactness in gradient arises, for example, in many applications such as optimization in infinite-dimensional spaces or gradient-free methods. Nevertheless, it is well-known that accelerated gradient first order methods can accumulate gradient error in theoretical estimates for the rate of convergence. In this work, we provide modificated well-known methods for convex optimization based on the subspace optimization. We research on the methods convergence for different condition of inexactness both in gradient value and accuracy of subspace optimization problems. Besides this, we investigate a generalization of this result to the class of quasar-convex (weakly-quasi-convex) functions.

keywords: subspace optimization, inexact gradient, non-convex optimization.
The research was supported by RSF, grant no 21-71- 30005.

Dmitry Perov[1] **and Oleg Kuzenkov**[1]

GLOBAL OPTIMIZATION METHOD BASED ON THE COMBINATION OF EVOLUTIONARY ALGORITHMS USING NEURAL NETWORKS

[1] Lobachevsky State University of Nizhny Novgorod, Russia
kuzenkov_o@mail.ru

There are many evolutionary algorithms for solving global multidimensional optimization problems. Each of them may get stuck in local extremes due to rapid convergence or spend many iterations wandering around a global extreme. This is the problem of exploration and exploitation. One of the solutions to this is a combination of algorithms which relies on the advantages and disadvantages of each algorithm. The goal of our work is to develop a new effective global optimization method based on a combination of evolutionary algorithms using neural networks. With the help of neural networks, we find out which of the following evolutionary methods can be effectively used in different stages of optimization: Differential Evolution (DE), Survival of the Fittest (SoFA), Covariance Matrix Adaptation (CMA), Novelty Search (NS). We trained the neural network on 100 randomly generated functions and tested it on relevant multidimensional functions. This approach has proven its effectiveness in solving global optimization problems compared to the algorithms considered individually.

keywords: evolutionary algorithms, neural networks, meta-optimization, global optimization.

Alexander Rogozin[1], **Anton Novitskii**[1] **and Alexander Gasnikov**[1,2,3]

DECENTRALIZED PROXIMAL OPTIMIZATION METHOD
WITH CONSENSUS PROCEDURE

[1] Moscow Institute of Physics and Technology, Russia
[2] Institute for Information Transportation Problems, Russia
[3] Caucasus Mathematical Center of Adygh State University, Russia
gasnikov.av@mipt.ru

Decentralized optimization is well studied for smooth unconstrained problems. However, constrained problems or problems with composite terms are an open direction for research. We study structured (or composite) optimization problems, where the functional is a sum of a convex smooth function and a proper convex proximal-friendly term. Our method builds upon an accelerated proximal gradient descent and makes several consensus iterations between computations.

keywords: convex optimization, distributed optimization, proximal method.

Alexander Strekalovsky[1]

ON CONVERGENCE OF GLOBAL SEARCH IN GENERAL
DC OPTIMIZATION PROBLEM

[1] Matrosov Institute for System Dynamics and Control Theory, Russia
strekal@icc.ru

We address the general DC optimization problem, where objective function, equality and inequality constraints are specified in terms of DC functions. First, with the help of the Exact Penalty Theory we reduce the considered problem to an auxiliary problem of unconditional optimization, whose objective function is a DC function, as well. Further, we prove the necessary and sufficient condition for minimizing sequence for such a penalized problem. On the basis of these foundations, we develop a 'theoretical' method for solving an auxiliary problem and prove its convergence. Using these results for solving the penalized problem, we develop a more realistic Global Search Scheme (GSS1) employing local search methods and classical optimization methods. Finally, with the help of so-called resolving sets we prove that the sequence, produced by GSS1 with resolving approximations, turns out to be minimizing for the penalized problem. Besides, under the feasibility condition, this is a minimizing sequence to the original problem with constraints.

keywords: DC optimization, optimality condition, global search method.

Demyan Yarmoshik[1], **Alexander Rogozin**[1] **and Alexander Gasnikov**[1,2,3]

DECENTRALIZED OPTIMIZATION WITH AFFINE CONSTRAINTS OVER TIME-VARYING GRAPHS

[1] Moscow Institute of Physics and Technology, Moscow, Russia
[2] Kharkevich nstitute for Information Transmission Problems, Russia
[3] Caucasus Mathematical Center of Adygh State University, Russia
gasnikov.av@mipt.ru

Decentralized optimization paradigm assumes that each term of the separated objective is privately stored by the corresponding agent. Agent are only allowed to communicate with their neighbours in the communication graph. We consider the case when the agents additionally have local affine constraints and the communication graph can change over time. We generalise optimal algorithm for time-varying decentralized optimization on the case of affine constraints and analyze its convergence.

keywords: distributed optimization, convex optimization, constrained optimization, time-varying networks.

The work of Demyan Yarmoshik was supported by the program "Leading Scientific Schools" (grant no. NSh-775.2022.1.1). The research of Alexander Rogozin and Alexander Gasnikov was supported by the Ministry of Science and Higher Education of the Russian Federation (Goszadaniye) 075-00337-20-03, grant no 0714-2020-0005.

Nikita Yudin[1] **and Alexander Gasnikov**[1,2,3]

SCALED GRADIENT METHODS FOR FASTER CONVEX OPTIMIZATION

[1] Moscow Institute of Physics and Technology, Russia
[2] Kharkevich Institute for Information Transmission Problems, Russia
[3] Caucasus Mathematical Center of Adygh State University, Russia
gasnikov.av@mipt.ru

We present an extension of the family of algorithms, called gradient descent algorithms for convex minimization. Our introduced algorithms are essentially first-order in terms of oracle usage, although these algorithms are capable to retrieve high-order information from the target function landscape to adjust each scaled gradient update eventually achieving better steps than simple gradient descent. We provide several examples of smooth functions and numerical experiments that verify our theoretical justifications even in the worst-case scenario.

keywords: convex optimization, high-order methods, tensor methods, first-order methods, global convergence.

4. Discrete and Combinatorial Optimization

Ivan Davydov[1], **Igor Vasilyev** [2,3] **and Anton V. Ushakov**[2]

TABU SEARCH METAHEURISTIC FOR THE PENALTY MINIMIZATION PERSONNEL TASK SCHEDULING PROBLEM

[1] Sobolev institute of Mathematics, Russia
[2] Matrosov Institute for System Dynamics and Control Theory, Russia
[3] Novosibirsk Research Center, Huawei Russian Research Institute, Russia
iadavydov@math.nsc.ru, {vil,aushakov}@icc.ru

Personnel scheduling is an active research field motivated by not only economic considerations but also the understanding of importance of improving working conditions and fairness in assigning employees to tasks. A large number of daily tasks and an expanding staff require effective automation of the task allocation process. In the problem under consideration, given sets of tasks and staff, it is required to assign the certain number of employees to each task, taking into account their skills. The goal is to minimize penalties induced by conflicting assignments as well as by uneven workload of the staff. To solve the problem, a two-phase heuristic consisting of a greedy heuristic followed by a randomize tabu search has been developed. Computational experiments shows that the proposed approach allows us to find optimal or near-optimal solutions on instances corresponding to real-life problems.

keywords: crew scheduling, personnel task scheduling, tabu search, MIP formulation, greedy algorithm, rostering.

Adil Erzin[1,2,3], **Roman Plotnikov**[1,3] **and Ilya Ladygin**[2]

AGGREGATION TREE CONSTRUCTION USING HIERARCHICAL STRUCTURES

[1] Sobolev institute of Mathematics, Russia
[2] Novosibirsk State University, Russia
[3] Saint Petersburg State University, Russia
{adilerzin, prv}@math.nsc.ru, ilia.lad@mail.ru

In the problem under consideration it is necessary to find a schedule of conflict-free data aggregation of a minimum length, i.e. to determine the transmission moments for each vertex of the communica- tion graph in such a way that there are no conflicts, and all data gets into the base station within the minimum time using the arcs of un- known spanning aggregation tree (AT). Although the problem remains NP-hard even with a known AT, finding an AT is an important step in conflict-free data aggregation. This paper proposes a new algorithm for constructing the ATs in the special hierarchical structures, k-HS, which initially may contain up to $k \geq 1$, copies of the same vertex located at k neighbouring levels. Then for different k we build a spanning tree k-HST and propose a heuristic to leave in k-HST a single copy of each node. The result is an aggregation tree k-AT. Using k-ATs constructed for different k, we find a conflict-free schedule applying the best-known algorithms. The outcome

of our method is the best schedule found on k-ATs with different values of k. To assess the quality of the proposed approach, we carried out a numerical experiment on the randomly constructed unit- disk graphs, and can conclude that the construction of k-ATs speeds up the aggregation.

keywords: hierarchical structure, aggregation tree, conflict-free aggregation, convergecast.

The research was supported by the RSF, grant No. 22-71-10063.

Adil Erzin[1], **Alexander Kononov**[1], **Stepan Nazarenko**[2] **and Konstantin Sharankhaev**[2]

AN $O(n \log n)$-TIME ALGORITHM FOR LINEARLY ORDERED PACKING OF 2-BAR CHARTS INTO $\mathrm{OPT} + 1$ BINS

[1] Sobolev institute of Mathematics, Russia
[2] Novosibirsk State University, Russia
{adilerzin,alvenko}@math.nsc.ru,
{s.nazarenko, k.sharankhaev}@g.nsu.ru

Given a sequence of bins and n bar charts consisting of two bars each (2-BCs). Every bar has a positive height not exceeding 1. Each bin can contain any subset of bars of total height at most 1. It is required to pack all 2-BCs into the minimal number of bins so that the bars of each 2-BC do not change their order and occupy adjacent bins. Previously, a special case of the problem was considered where the first bars of any two 2-BCs cannot be placed into the same bin. For this case an $O(n^2)$-time algorithm that constructs a packing of length at most $\mathrm{OPT} + 1$, where OPT is the optimum, was presented. In this paper, we propose a new, less time-consuming algorithm that also constructs a packing of length at most $\mathrm{OPT} + 1$ for the same case of the problem with time complexity equals to $O(n \log n)$.

keywords: bar charts, packing, approximation.

Dmitry Gribanov[1,2]

ENUMERATION AND UNIMODULAR EQUIVALENCE OF EMPTY DELTA-MODULAR SIMPLICES

[1] Lobachevsky State University of Nizhny Novgorod, Russia
[2] National Research University Higher School of Economics, Russia
dimitry.gribanov@gmail.com

Consider a class of simplices defined by systems $Ax \leq b$ of linear inequalities with Δ-*modular* matrices. A matrix is called Δ-*modular*, if all its rank-order sub-determinants are bounded by Δ in an absolute value. In our work we call a simplex Δ-*modular*, if it can be defined by a system $Ax \leq b$ with a Δ-modular

matrix A. And we call a simplex *empty*, if it contains no points with integer coordinates. In literature, a simplex is called *lattice*, if all its vertices have integer coordinates. And a lattice-simplex is called *empty*, if it contains no points with integer coordinates excluding its vertices. Recently, assuming that Δ is fixed, it was shown in `http://dx.doi.org/10.1007/s10898-022-01165-9` that the number of Δ-modular empty simplices modulo the unimodular equivalence relation is bounded by a polynomial on dimension. We show that an analogous fact holds for the class of Δ-modular empty lattice-simplices. As the main result, assuming again that the value of the parameter Δ is fixed, we show that the all unimodular equivalence classes of simplices of both types can be enumerated by a polynomial-time algorithm. As the secondary result, we show the existence of a polynomial-time algorithm for the problem to check the unimodular equivalence relation for a given pair of Δ-modular (not necessarily empty) simplices.

keywords: lattice simplex, empty simplex, delta-modular matrix, bounded sub-determinants, unimodular equivalence, enumeration algorithm.

The article was prepared under financial support of RSF grant no 21-11-00194.

Victor Il'ev[1,2] **and Svetlana Il'eva**[1]

APPROXIMATION ALGORITHMS FOR GRAPH CLUSTER EDITING PROBLEMS WITH CLUSTERS OF BOUNDED SIZES

[1] Dostoevsky Omsk State University, Russia

[2] Sobolev Institute of Mathematics, Russia

`iljev@mail.ru`

In clustering problems, one has to partition a given set of objects into pairwise disjoint subsets (clusters) taking into account only similarity of objects. In the graph cluster editing problem similarity relation on the set of objects is given by an undirected graph whose vertices are in one-to-one correspondence with objects and edges correspond to pairs of similar objects. The goal is to find a nearest to a given graph $G = (V, E)$ cluster graph, i.e., a graph on the vertex set V each connected component of which is a complete graph. The distance between graphs is understood as the Hamming distance between their incidence vectors. We consider a variant of the cluster editing problem in which the size of each cluster is bounded from above by a positive integer s. In 2011, Il'ev and Navrotskaya proved that this problem is NP-hard for any fixed $s \geqslant 3$. In 2015, Puleo and Milenkovic proposed a 6-approximation algorithm for this problem. In 2016, Il'ev, Il'eva and Navrotskaya presented an approximation algorithm that is 3-approximation in case of $s = 3$ and 5-approximation in case of $s = 4$. Now we propose simple greedy-type 2-approximation algorithms for these cases with tight performance guarantees.

keywords: graph, cluster editing, approximation algorithm, performance guarantee.

Vladimir Khandeev[1] **and Sergey Neshchadim**[2]

CONSTANT-FACTOR APPROXIMATION ALGORITHMS FOR SOME MAXIMIN MULTI-CLUSTERING PROBLEMS

[1] Sobolev Institute of Mathematics, Russia
[2] Novosibirsk State University, Russia
khandeev@math.nsc.ru, s.neshchadim@g.nsu.ru

We consider several problems of searching for a family of non-intersecting subsets in a finite set of points of Euclidean space. In these problems, it is required to maximize the minimum cluster's cardinality under constraint on each cluster's scatter. The scatter is the sum of the distances from the cluster elements to the center, which is defined differently in each of the problems. We show that these problems are NP-hard in the case of an arbitrary fixed number of clusters and propose polynomial constant-factor approximation algorithms for this case.

keywords: Euclidean space, clustering, max-min problem, NP-hardness, bounded scatter, approximation algorithm.

Andrei V. Nikolaev[1] **and Alexander V. Korostil**[2]

ON CONE PARTITIONS FOR THE MIN-CUT AND MAX-CUT PROBLEMS WITH NON-NEGATIVE EDGES

[1] Demidov Yaroslavl State University, Russia
[2] Tinkoff Bank, Russia
{andrei.v.nikolaev, av.korostil}@gmail.com

We consider the classical minimum and maximum cut problems: find a partition of vertices of a graph into two disjoint subsets that minimize or maximize the sum of the weights of edges with endpoints in different subsets. It is known that if the edge weights are non-negative, then the min-cut problem is polynomially solvable, while the max-cut problem is NP-hard. We construct a partition of the positive orthant into convex cones corresponding to the characteristic cut vectors, similar to a normal fan of a cut polyhedron. A graph of a cone partition is a graph whose vertices are cones, and two cones are adjacent if and only if they have a common facet. We define adjacency criteria in the graphs of cone partitions for the min-cut and max-cut problems. Based on them, we show that for both problems the vertex degrees are exponential, and the graph diameter equals 2. These results contrast with the clique numbers of graphs of cone partitions, which are linear for the minimum cut problem and exponential for the maximum cut problem.

keywords: min-cut and max-cut problems, cut polytope, cone partition, 1-skeleton, vertex adjacency, graph diameter, vertex degree, clique number.

Artem V. Pyatkin[1]

PTAS FOR P-MEANS Q-MEDOIDS R-GIVEN CLUSTERING PROBLEM

[1] Sobolev Institute of Mathematics, Russia
artempyatkin@gmail.com

We consider the problem of clustering (partitioning) a given set of d-dimensional Euclidean vectors into a fixed number of clusters with different center types so that the total dispersion would be minimal, where by dispersion we mean the sum of squared distances from the elements of the cluster to its center. We consider the case where p centers are means (or centroids) of the cluster elements, q centers are medoids (some vectors from the initial sets) and r centers are given (fixed points of the space). We assume that the sizes of the clusters are also given in advance. It is known that htis problem is NP-hard even for $p \geq 1$ and $p+q+r \geq 2$. In this paper a PTAS of complexity $O((d+n)n^{q+2+p/\varepsilon})$ for this problem is suggested.

keywords: clustering, k-means, centroid, medoid, PTAS.

Alexey Ratushnyi

A PATTERN-BASED HEURISTIC FOR A TEMPORAL BIN PACKING PROBLEM WITH CONFLICTS

[1] Sobolev Institute of Mathematics, Russia
alexeyratushny@gmail.com

We introduce a new temporal bin packing problem with conflicts that originated from cloud computing. We have a finite set of items and a time horizon. For each item, we know an arriving time and a finish time from the horizon. Moreover, each item has two weights (CPU cores and RAM) which are defined by its type. The number of types is much smaller than the number of items. Each bin (server) has two capacities and is divided into several non-identical parts called NUMA nodes. Some items are large. They have to be split into two predefined parts and packed into two different nodes. A regular item can be packed without any split. Our goal is to pack all items into the minimum number of identical bins during the time horizon. For this NP-hard problem, we design a heuristic based on a column generation approach to get lower and upper bounds. We carry out computational experiments on semi-synthetic instances with 10 types, 10000 items for bins with 2 NUMA nodes, and 20000 items for bins with 4 NUMA nodes. Every instance has a 10000 long time horizon. Computational results show the effectiveness of the approach and a small average gap.

keywords: conflicts, bin packing problem, temporal, column generation, column generation, knapsack problem.

Artem V. Ripatti[1]

NESTED (2,3)-INSTANCES OF THE CUTTING STOCK PROBLEM

[1] Ufa University of Science and Technology, Russia

ripatti@inbox.ru

We consider the well-known cutting stock problem (CSP). A $(2,3)$-instance of CSP is an instance for which each of its items can only be packed into the container either two or three times. The gap of a CSP instance is the difference between its optimal function value and the optimal value of its continuous relaxation. For most instances of CSP the gap is less than 1, and the maximal gap currently known is $77/64 = 1.203125$. For the $(2,3)$- case the best known construction gives the gap greater than $73/72 - \varepsilon \approx 1.013888$ for any $\varepsilon > 0$. In this paper we give a construction of $(2,3)$-instances having recursive (nested) structure and the gap greater than $7/6 - \varepsilon \approx 1.166666$ for any $\varepsilon > 0$. We also give a construction of CSP instances with the gap greater than $59/48 - \varepsilon \approx 1.229166$ for any $\varepsilon > 0$.

keywords: cutting stock problem, integer round up property, integrality gap.

Ruslan Yu. Simanchev[1,2] **and Inna V. Urazova**[1]

INTEGER MODELS FOR THE TOTAL WEIGHTED TARDINESS PROBLEM ON A SINGLE MACHINE

[1] Dostoevsky Omsk State University, Russia

osiman@rambler.ru, urazovainn@mail.ru

The paper deals with the following scheduling problem. Jobs are served by a single machine. Each job is characterized by a positive weight, a release date, a due date. Servicing times are the same for all jobs. Preemptions are allowed. We investigate the case of the discrete time. The goal is to find a schedule for servicing jobs that minimizes the weighted sum of tardiness. The complexity status of this problem is unknown today. Our paper proposes two models of boolean linear programming for this problem. A comparative analysis of the models is carried out, and results of the computational experiment are described.

keywords: schedules, tardiness, polyhedral approach, valid inequalities, (0,1)-programming.

This work was carried out within the governmental order for Omsk Scientific Center SB RAS (project registration number 121022000112-2)

Gennady Zabudsky[1]

SOLVING MAXIMIN LOCATION PROBLEMS ON NETWORKS WITH DIFFERENT METRICS AND RESTRICTIONS

[1] Sobolev Institute of Mathematics, Russia
zabudsky@ofim.oscsbras.ru

Several optimal location problems of an obnoxious facility on a network of roads connecting settlements are considered. It is necessary to find such location of the facility so that a minimum distance to a nearest settlement is as large as possible taking into account the resident population. Such facility can be, for example, a nuclear power plant, a waste recycling plant. An overview of various formulations, the properties of the problems and algorithms for solving are given. The main focus is on the problem taking into account a restriction on transportation costs for servicing the settlements by the facility. The cost of servicing the settlements by the facility is determined using the shortest paths in the network. The objective function uses Euclidean metric. Exact algorithm for solving of this problem is proposed.

keywords: Euclidean metric, max-min criterion, obnoxious facility, shortest paths, Voronoi diagram.

The research was funded in accordance with the state task of the IM SB RAS, project FWNF-2022-0020

Saeid Alikhani[1], **Hamidreza Golmohammadi**[2] **and Elena Konstantinova**[2,3]

SOME RESULTS ON THE COALITION NUMBER OF CUBIC GRAPHS OF ORDER AT MOST 10

[1] Yazd University, Iran
[2] Novosibirsk State University, Russia
[3] Sobolev institute of Mathematics, Russia
alikhani@yazd.ac.ir, h.golmohammadi@g.nsu.ru, e_konsta@math.nsc.ru

A coalition in a graph G consists of two disjoint sets of vertices V_1 and V_2, neither of which is a dominating set but whose union $V_1 \cup V_2$, is a dominating set. A coalition partition in a graph G is a vertex partition $\pi = V_1, V_2, \ldots, V_k$ such that every set $V_i \in \pi$ is not a dominating set but forms a coalition with another set $V_j \in \pi$ which is not a dominating set. The coalition number $C(G)$ equals the maximum k of a coalition partition of G. The class of cubic graphs is especially interesting for mathematical applications, because for various important open problems in graph theory, cubic graphs are the smallest or simplest possible potential counterexamples, and so this creates motivation to study coalition parameter for the cubic graphs of order at most 10.

keywords: coalition, cubic graphs, Petersen graph.

Aleksandr Bulavchuk[1] **and Daria Semenova**[1]

Using hierarchical copulas when solving RCPSP

[1] Siberian Federal University, Russia
DVSemenova@sfu-kras.ru

The paper considers the resource-constrained project scheduling problem with the maximizing of net present value. The components of the project's cash flows are random variables with known distribution. The project is financed, among other things, by reinvesting the profits received. To model project risks, it is proposed to use hierarchical copulas. This approach allows taking into account the mutual dependence of the random parameters of the project. To demonstrate the approach to risk assessment, computational experiments were carried out to find optimal schedules. To solve the problem, the genetic algorithm GASPIA and the annealing simulation method previously proposed by the authors were used.

keywords: scheduling problem, investment project, NPV, genetic algorithm, simulated annealing. hierarchical copulas.

Evgenii Burashnikov[1]

Branch-and-bound and dynamic programming approaches for the knapsack problem

[1] Higher School of Economics, Russia
epburashnikov@hse.ru

The knapsack problem is one of the most popular NP-hard problems in combinatorial optimization. For 0-1 Knapsack Problem there are two famous approaches which guarantee the optimality of the solutions: Branch-and-Bound (BnB) and Dynamic Programming (DP) algorithms. For DP we suggest some pre-processing and rules which help us to avoid unneeded calculations. For BnB we provide an algorithm which uses the same cache of solutions as DP does but fills only the necessary part of this cache and does it gradually during the search. Computational experiments on artificially generated data and com-mon benchmarks show the effectiveness of the proposed modifications.

keywords: knapsack 0-1 problem, branch-and-bound, dynamic programming, combinatorial optimization.

Aleksander Chirkov[1], **Dmitry Gribanov**[1,2], **Sergey Veselov**[1] **and Nikolai Zolotykh**[1]

On the number of edges in integer polyhedra

[1] Lobachevsky State University of Nizhny Novgorod, Russia

[2] National Research University Higher School of Economics, Russia
dimitry.gribanov@gmail.com

Let $P(\Lambda, b)$ be the convex hull of all non-negative points in $b+\Lambda$, where $b \in \mathbf{Z}^n$, $\Lambda \subseteq \mathbf{Z}^n$, and Λ is a sub-lattice of $\mathbf{Z}^n$. Denote by Δ the determinant of Λ. It is well-known, that the number of vertices in $P(\Lambda, b)$ is at most $O(\log^{n-1} \Delta)$, if n is fixed, and this upper bound is tight. On the base of this bound, it is easy to obtain the upper bound for the number of edges, $O(\log^{2n-2} \Delta)$. We improve this bound and show that the number of edges in $P(\Lambda, b)$ is at most $O(\log^{2n-3} \Delta)$. This is the first advance in the last 30 years in the problem of establishing a tight bound for the number of the edges (as well as for the number of faces of any other given dimension except for vertices), which is apparently $O(\log^{n-1} \Delta)$.

keywords: lattice, integer polyhedron, integer points, convex hull, vertices, edges, faces.

Andrey Dobrynin[1]

On the preservation of the Wiener index upon removing a vertex in a cubic graph

[1] Sobolev Institute of Mathematics, Russia
dobr@math.nsc.ru

The Wiener index, $W(G)$, is the sum of distances between all vertices of a connected graph G. In 2018, Majstorović, Knor and Skrekovski posed the problem of finding r-regular graphs except the cycle of order 11 having at least one vertex with property $W(G) = W(G–v)$. An infinite family of cubic graphs with four such vertices is presented.

keywords: Wiener index, regular graph.

Anton Eremeev[1], **Vyacheslav Ustyugov**[1] **and Lidia Zaozerskaya**[1]

On local search for MILP solver parameters optimization

[1] Sobolev Institute of Mathematics, Russia
eremeevtmp@yandex.ru, zaozer@ofim.oscsbras.ru

Mixed Integer Linear Programming (MILP) solvers, such as GUROBI, CPLEX, etc., besides their problem input may also receive a set of tunable parameters that affect the performance of such solvers. The algorithm ParamILS (Hutter et al, 2009) implements iterated local search over the parameter space of a solver, which is applied to multiple MILP instances from some problem-specific domain. ParamILS may be viewed as a simplified Variable Neighborhood Search (VNS) algorithm (Hansen, P.; Mladenović, 1997) with a single neighborhood type, combined with a special method of balancing exploration vs exploitation.

In this work, we develop a VNS method with multiple neighborhood types and the same method of balancing exploration vs exploitation as in ParamILS. Both algorithms are tested and compared on publicly available MILP benchmarks and instances of the Rig Routing Problem (Kulachenko, Kononova, 2020).

keywords: VNS, ParamILS, iterated local search, parameter optimization.

Yana Glotova[1]

A local search heuristic for the temporal knapsack problem

[1] Sobolev Institute of Mathematics, Russia
i.glotova@alumni.nsu.ru

We have a set of virtual machines (VM) and a server. For each VM, we know the CPU and RAM capacities, profit, and time interval when the VM is active. All VMs are divided into large and small. The server has a structure with two NUMA-nodes. Large VMs are divided into half and located on both nodes, while small ones are placed on one of the nodes. Our goal is to find a subset of VMs and their location on the server to maximize the total profit under the server's CPU and RAM capacity constraints during the whole-time interval. To solve this NP-hard problem, the adaptive large neighborhood search method is developed based on some "destroy" and "repair" heuristics. At the "destroy" stage, some items are removed from the current solution. At the "repair" stage, a new solution is created based on the greedy and local improvement heuristics. For the initial solution, we apply the greedy algorithm. Computational experiments are conducted for the synthetic randomly generated instances with up to 500 VMs. Computational results are discussed and compared with near-optimal solutions of commercial software Gurobi.

keywords: local search, NP-hard problem, temporal knapsack problem.

Daniil Khachai[1], **Olga Battaïa**[1],
Ruslan Sadykov[2] **and Michael Khachay**[3]

PRECEDENCE CONSTRAINED GENERALIZED TRAVELING SALESMAN PROBLEM: POLYHEDRAL ANALYSIS, MILP MODELS AND BRANCH-AND-CUT ALGORITHM

[1] KEDGE Business school, France
[2] Inria Centre de recherche Bordeaux Sud-Ouest, France
[3] Krasovsky Institute of Mathematics and Mechanics, Russia
{daniil.khachai, olga.battaia}@kedgebs.com, ruslan.sadykov@inria.fr, mkhachay@imm.uran.ru

The Precedence Constrained Generalized Traveling Salesman Problem (PCGTSP) is an extension of two well-known combinatorial optimization problems: the Generalized Traveling Salesman Problem (GTSP) and the Precedence Constrained Asymmetric Traveling Salesman Problem (PCATSP), whose path version is known as the Sequential Ordering Problem (SOP). Similarly to the classic GTSP, the goal of the PCGTSP, for a given input digraph and partition of its node set into clusters, is to find a minimum cost cyclic route (tour) visiting each cluster in a single node. In addition, as in the PCATSP, feasible tours are restricted to visit the clusters with respect to the given partial order. Unlike the GTSP and SOP, to the best of our knowledge, the PCGTSP still remain to be weakly studied both in terms of polyhedral theory and algorithms. In this paper, for the first time for the PCGTSP, we propose several families of valid inequalities, establish dimension of the PCGTS poly- tope and prove sufficient conditions ensuring that the extended Balas' π- and σ-inequalities become facet-inducing. Relying on these theoretical results and evolving the state-of-the-art algorithmic approaches for the PCATSP and SOP, we introduce a family of MILP-models (formulations) and several variants of the branch-and-cut algorithm for the PCGTSP. We prove their high performance in a competitive numerical evaluation against the public benchmark library PCGTSPLIB, a known adaptation of the classic SOPLIB to the problem in question.

keywords: TSP, PCGTSP, SOP, branch-and-cut, integer programming.

Yaroslav A. Kharchenko and Alexander V. Kononov

PREDICTION-AUGMENTED ALGORITHMS FOR PARKING PERMIT PROBLEM

[1] Sobolev Institute of Mathematics, Russia
kharchenko.yar@gmail.com, alvenko@mail.math.nsc.ru

We consider the online parking permit problem that is the natural generalization of the well-studied ski rental problem. Let's assume that on certain days we drive to work by car. On any driving days, we must obtain a valid parking permit. Whether we go by car or on foot depends on many factors, such as

the weather, and it is not known in advance. Permits have different costs and are valid for a different number of days. Longer duration permits tend to cost less per day. If we drive every day, it is advantageous to take a permit for the longest period. If we drive only occasionally, it is more profitable to buy short duration permits.

keywords: online, competitive ratio, learning augmented, parking permit problem, consistency, robustness.

Polina Kononova[1] **and Igor Kulachenko**[1]

A decomposition approach for solving a large-scale consistent vehicle routing problem.

[1] Sobolev Institute of Mathematics, Russia
pkononova@math.nsc.ru, soge.ink@gmail.com

A large-scale consistent vehicle routing problem is under consideration. In the problem, a company has a fleet of capacitated vehicles located in multiple depots, and there is a set of customers requiring a service. Vehicle routing is performed on a given planning horizon consisting of some number of days, and a vehicle should not exceed a limit on working hours per day. We have a frequency of visits for each customer, which is how many times the customer must be visited during the planning horizon. Besides, to ensure consistency in the schedule, a customer needs to be visited by the same vehicle, and the time intervals between two consecutive visits must be the same. This allows the company to gain a competitive advantage by improving its service quality. The objective of the problem is to minimize total travel costs while satisfying all the constraints. We present a mixed-integer linear programming model for the problem. To enable solving large-scale instances of the problem, we employed a POPMUSIC-based decomposition approach. The idea behind this approach is to improve the quality of a solution by iteratively solving subproblems of an overall problem. To get a solution for the overall problem and its subproblems, we use both Gurobi solver and the variable neighborhood search (VNS) scheme developed by us previously. In the computational experiments section, we show a comparison of applying VNS for the whole problem and applying different implementations of the decomposition approach.

keywords: capacitated vehicles, multiple depots, periodic, variable neighborhood search.

Mikhail Posypkin[1]

A heuristic for the bin packing problem.

[1] Federal Research Center for Computer Science and Control, Russia
mposypkin@frccsc.ru

The bin packing problem is a well-known combinatorial problem consisting in finding the minimal number of equally sized bins to accommodate the given set of items with known weights. In spite of its simple formulation, the problem is NP-complete, and exact methods may fail even for moderate-size instances. Thus, heuristic methods become a tool of choice in practice. In this paper, we propose a new heuristic combining reductions, partial enumerations, and greedy algorithms. We compare the performance of the proposed heuristic with that of classical algorithms. An experimental study has demonstrated the superiority of the new algorithm with respect to the existing approaches for a significant portion of the benchmark problems.

keywords: bin packing problem, heuristic methods.

Ksenia Rizhenko[1], **Katherine Neznakhina**[1] **and Michael Khachay**[1]

A polynomial-time constant-ratio approximation algorithm for asymmetric generalized traveling salesman problem.

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
{k.v.rizhenko, mkhachay}@imm.uran.ru, eneznakhina@yandex.ru

We concider the Asymmetric Generalized Traveling Salesman Problem (AGTSP). An instance of the AGTSP is given by an edge-weighted directed graph $G = (V, E, c)$ and partition of its node set into several clusters. The goal is to find a minimum weight closed route visiting each cluster in a single node. We propose a first polynomial-time constant-ratio approximation algorithm for AGTSP. Our approach extends the framework proposed by Svensson and Traub for the Asymmetric Traveling Salesman Problem (ATSP).

keywords: polynomial-time approximation algorithm, constant-ratio approximation, Asymmetric Generalized Traveling Salesman Problem.

Alexandr Seliverstov[1]

Some obstacles to dimensionality reduction in pseudo-Boolean programming

[1] Kharkevich Institute for Information Transmission Problems, Russia
slvstv@iitp.ru

We consider the arrangement of vertices of the unit multidimensional cube and affine subspace as well as their orthographic projections onto the coordinate

subspace. Upper and lower bounds on the subspace dimension are given under which some orthographic projection always preserves the incidence relation between the subspace and cube vertices. The proved upper bound is equal to the integer part of half the dimension of the ambient space. The proved lower bound is logarithmic. Our new results are obtained by both analytic and synthetic methods. They illustrate the complexity of pseudo-Boolean programming problems because the reduction of the dimension of the problem by orthographic projection meets obstacles in the worst case.

keywords: multidimensional cube, affine subspace, projection, incidence geometry, discrete optimization.

Sofia Shperling[1] **and Yury Kochetov**[2]

A FITNESS FUNCTIONS BASED ALGORITHM FOR THE TWO-DIMENSIONAL IRREGULAR STRIP PACKING PROBLEM

[1] Novosibirsk State University, Russia
[2] Sobolev institute of Mathematics, Russia
jkochet@math.nsc.ru

We are given a finite set of items (irregular polygons) and an infinite strip with known width. The goal is to pack all items without overlapping so to minimize the used length of the strip. Rotation of items by 180° are allowed. We apply the well-known semi-discrete representation of the items in the strip. The idea is to represent each item as a series of equidistant horizontal line segments. For given item permutation, our algorithm packs items one-by-one and uses some fitness functions based on the gravity center of partial packing, the parameters of the packing, and semi-discrete item representation. The algorithm detects the next bottom-left empty place in the partial packing and selects the best unpacking item for it. The fitness functions calculate for the selected place and for all items from a sliding window with random size. The item with the lowest total value of the fitness functions is packed in the empty place. To find the best item permutation, we apply the randomized local search approach with 2-opt neighborhood. Computational results for available test instances with up to 64 items are discussed.

keywords: Nesting problem, semi-discrete representation, fitness function, Local search.

Aleksey Zakharov[1]

A METAHEURISTIC WITH OPTIMIZED OPERATORS FOR PROBLEMS WITH TREE-BASED SOLUTION REPRESENTATION

[1] Sobolev institute of Mathematics, Russia
zakh.alexey@gmail.com

We propose a new population-based algorithm for problems allowing tree-based

solution representation. The important characteristic of the proposed algorithm is optimized operators, based on solving subproblems of the reduced dimension. The computational complexity of the operators is analyzed. The computational experiment was carried out on instances from the known libraries. We also apply strategies for adopt turning of algorithm parameters. The obtained results can be used for improving of program code or classification of operations in applications for computer systems.

keywords: tree, local search, adaptation, optimization.

The research was supported by RSF, grant no 22-71-10015.

Lidia Zaozerskaya[1]

GENETIC ALGORITHM FOR THE GENERALIZED ASSIGNMENT PROBLEM WITH ADDITIONAL CONSTRAINTS

[1] Sobolev institute of Mathematics, Russia

zaozer@ofim.oscsbras.ru

To solve the generalized assignment problem with additional restrictions on the minimum load of agents and upper restrictions on the number of job types assigned to agents, a genetic algorithm with optimal recombination has been developed. The results of a comparative analysis of the operation of the algorithm with the Gurobi solver and previously proposed matheuristic based on local search are presented.

keywords: generalized assignment problem, genetic algorithm, Gurobi solver, matheuristic.

5. Stochastic Optimization

Maxim Ershov[1] **and Albert Voroshilov**[2]

UCB strategy for Gaussian and Bernoulli multi-armed bandits

[1] Yaroslav-the-Wise Novgorod State University, Russia
s244528@std.novsu.ru

We have considered a modification of the UCB strategy for a multi-armed bandit having a Gaussian or Bernoulli distribution of one-step incomes. This strategy involves choosing an action that corresponds to the current highest value of the upper confidence bound (UCB) of the interval estimates of mathematical expectations of one-step income. The control goal is a minimax strategy, that means in minimizing maximum regrets. We computed the maximum regrets using Monte-Carlo simulations. We also performed a regression analysis of the function of the dependence of the maximum of the regret function on the strategy parameter.

keywords: multi-armed bandit, UCB Strategy, min-max approach, regression analysis.. Supported by RSF, project no 23-21-00447, https://rscf.ru/en/project/23-21-00447/

Sergey Garbar[1]

Estimation of both unknown parameters in Gaussian multi-armed bandit for batch processing scenario

[1] Yaroslav-the-Wise Novgorod State University, Russia
Sergey.Garbar@novsu.ru

We consider a Gaussian multi-armed bandit problem with both reward means and variances unknown. A Gaussian multi-armed bandit is considered because in case of batch processing the cumulative rewards for the batches are distributed close to normally. A batch version of the UCB strategy is proposed. Strategy's description that is invariant in regards to the horizon size is obtained. We consider different approaches to the task of estimating unknown variances of rewards and study their effect on the normalized regret. A set of Monte-Carlo simulations is performed to study the batch strategy and illustrate the results for the two-armed bandit.

keywords: multi-armed bandit, two-armed bandit, Gaussian multi-armed bandit, UCB, batch processing, Monte Carlo simulation.. Supported by RSF, project no 23-21-00447, https://rscf.ru/en/project/23-21-00447/

Aleksei Ignatov[1]

On the resource allocation problem to increase reliability of transportation systems

[1] Moscow Aviation Institute, Russia
alexei.ignatov1@gmail.com

The resource allocation problem to increase reliability of transportation sys-

tems is considered in the paper. We use probabilities of various undesirable events to describe reliability of transport system ele- ments. Probabilistic and quantile criteria are considered for optimization. The problem with probabilistic criterion is reduced to the integer linear programming problem. We propose the procedure to obtain an approximate solution in the problem with the quantile criterion based on the Chernoff bound and solution of mixed integer nonlinear programming problems.

keywords: reliability, transportation systems, probability, transportation, quantile, Chernoff bound, integer programming..

Aleksandr Lobanov[1,2,3], **Anton Anikin**[4], **Alexander Gasnikov**[1,5,6], **Alexander Gornov**[4], **Sergey Chukanov**[7]

ZERO-ORDER STOCHASTIC CONDITIONAL GRADIENT SLIDING METHOD FOR NON-SMOOTH CONVEX OPTIMIZATION

[1] Moscow Institute of Physics and Technology, Russia
[2] ISP RAS Research Center for Trusted Artificial Intelligence, Russia
[3] Moscow Aviation Institute, Russia
[4] Matrosov Institute for System Dynamics and Control Theory, Russia
[5] Kharkevich Institute for Information Transmission Problems, Russia
[6] Caucasus Mathematical Center, Adyghe State University, Russia
[7] Federal Research Center Computer Science and Control, Russia
{lobanov, gasnikov}@mipt.ru, {anikin,gornov}@icc.ru, chukanov47@mail.ru

The conditional gradient idea proposed by Marguerite Frank and Philip Wolfe in 1956 was so well received by the community that new algorithms (also called Frank–Wolfe type algorithms) are still being actively created. In this paper, we study a non-smooth stochastic convex optimization problem with constraints. Using a smoothing technique and based on an accelerated batched first-order Stochastic Conditional Gradient Sliding method, we propose a novel gradient-free Frank–Wolfe type algorithm called Zero-Order Stochastic Conditional Gradient Sliding (ZO-SCGS). This algorithm is robust not only for the class of non-smooth problems, but surprisingly also for the class of smooth black box problems, outperforming the SOTA algorithms in the smooth case in term oracle calls. In practical experiments we confirm our theoretical results.

keywords: Frank-Wolfe type algorithms, non-smooth convex optimization, gradient-free method.. The work was supported by the Ministry of Science and Higher Education of the Russian Federation (Goszadaniye) 075-00337-20-03, project no 0714-2020-0005.

Konstantin Semenikhin and Alexandr Arkhipov

DISTRIBUTIONALLY ROBUST OPTIMIZATION BY PROBABILITY CRITERION FOR ESTIMATING A BOUNDED SIGNAL

[1] Moscow Aviation Institute, Russia
siemenkv@mail.ru, ege3145@yandex.ru

This paper aims at solving a distributionally robust minimax estimation problem to recover a bounded smooth signal from the finite number of measurements with known second-order moment characteristics of the observation noise. The objective functional is the probability that the L2-norm of the estimation error will exceed a given threshold. To take into account the prior uncertainty, the upper bound of the probability functional is considered over the family of noise distributions and the set of signals with bounded second derivative. The goal of the problem is to minimize the worst-case error probability over the class of linear estimators. A specific feature of this problem is a major significance of the bias and its guaranteed bound. To solve the robust optimization problem with the probability objective we follow two methods: 1) direct minimization of the MSE-bound derived from the Markov inequality; 2) applying the explicit multivariate Selberg bound to the problem with quantile criterion. Numerical simulations are performed to compare the two methods applied to the problem of target path recovery.

keywords: min-max estimation, distributionally robust optimization, error probability, bounded signal, multivariate Selberg bound, cvx.

Sergey V. Ivanov, Andrey I. Kibzun and Valentina N. Akmaeva

ALGORITHM FOR FINDING A GUARANTEED SOLUTION TO A QUANTILE OPTIMIZATION PROBLEM WITH CONVEX LOSS FUNCTION

[1] Moscow Aviation Institute, Russia
siemenkv@mail.ru, ege3145@yandex.ru

We consider a stochastic programming problem with quantile criterion.The loss function is assumed to be piecewise linear in random parameters and convex in decision variables. The distribution of the random parameters is normal. By using the confidence method, we obtain convex approximating problems for finding lower and upper bounds of the optimal objective value. These problems are parameterized by the radius of a ball inscribed in a confidence polyhedral set. We suggest an algorithm for finding the radius of a ball that provides a guaranteed solution to the problem. This algorithm allows us to improve the initial upper bound. We study the accuracy of the obtained lower and upper bounds.

keywords: stochastic programming, quantile criterion.

The work was supported by the RSF, project no 22-21-00213.

Alexander Kolnogorov[1], **Maxim Ershov**[1] **and Albert Voroshilov**[1]

CUSTOMIZATION OF THE AUER–CESA-BIANCHI–FISCHER UCB STRATEGY FOR A GAUSSIAN ONE-ARMED BANDIT

[1] Yaroslav-the-Wise Novgorod State University, Russia

{s244525, s244528}@std.novsu.ru

We consider the one-armed bandit problem as applied to batch data processing if there are two processing methods with a priori known efficiency of the first method. For example, the efficiency is the probability of successful processing of a data unit and incomes are successfully processed data. The goal of the control is to maximize the total expected income, i.e., the total number of successfully processed data. During data processing, one has to determine the most effective method and provide its preferential use. Since the data is processed in batches and cumulative incomes of batches are used for the control, their distributions are approximately Gaussian. For a Gaussian one-armed bandit we investigate two cases, namely, of a priori known and unknown variance of the income corresponding to the method with unknown efficiency. The case of a priori known variance occurs, e.g., if the batch sizes are large enough, the variance is estimated during processing the first batch and then this estimate is used for the control. In the case of a priori unknown variance, it is assumed that the batch sizes are moderate ones and the variance is estimated during control. For the control, we customize the UCB strategy first proposed by Auer, Cesa-Bianchi and Fisher and investigate minimax goal of the control. In both cases of a priori known and unknown variance we obtain invariant descriptions of the control on the horizon equal to one. These descriptions allow us to determine the optimal parameters of the strategies by Monte-Carlo simulations. Numerical results show the high efficiency of the proposed UCB strategies.

keywords: one-armed bandit, UCB strategy, min-max approach, batch processing, invariant description.. Supported by RSF, project no 23-21-00447, https://rscf.ru/en/project/23-21-00447/.

Aleksandr Lobanov[1,2,3] **and Alexander Gasnikov**[1,4,5]

ZERO-ORDER ALGORITHMS FOR SOLVING NON-SMOOTH STOCHASTIC OPTIMIZATION PROBLEMS WITH HEAVY-TAILED NOISE

[1] Moscow Institute of Physics and Technology, Russia

[2] ISP RAS Research Center for Trusted Artificial Intelligence, Russia

[3] Moscow Aviation Institute, Russia

[4] Kharkevich Institute for Information Transmission Problems, Russia

[5] Caucasus Mathematical Center, Adyghe State University, Russia

{lobanov, gasnikov}@mipt.ru

In this paper, we study a non-smooth stochastic optimization problem (black-box problem) with heavy-tailed noise. Using the smoothing technique via L_2-

randomization, we replace the original non-smooth problem with a smooth one. Then based on the first-order optimal methods, we propose to solve the initial problem for the convex case by Zero-Order Clipped Stochastic Similar Triangles Method (ZO-clipped-SSTM), and for the convex-concave case by Zero-Order Clipped Stochastic Extragradient (ZO-clipped-SEG). In our analysis, we use two assumptions on variance, namely, when variance is bounded and when it is not. And in practical experiments we demonstrate the effectiveness of gradient clipping for non-smooth problems.

keywords: Gradient-free methods, clipping, heavy-tailed noise, L_2-randomization.

Alexey Tikhomirov[1]

On the convergence rate of the simple Markov homogeneous random search algorithm

[1] Yaroslav-the-Wise Novgorod State University, Russia

tikhomirov.as@mail.ru

Consider the problem of finding the global maximum of an objective function $f\colon \mathbb{R}^d \to \mathbb{R}$. One possible approach to this problem is to apply random search optimization methods. This paper is devoted to the theoretical study of the convergence rate of the simple Markov homogeneous random search algorithm. Suppose that the objective function $f\colon \mathbb{R}^d \to \mathbb{R}$ takes its maximal value at a single point x_*. We use a random search to find the maximizer x_* with the prescribed accuracy ε (approximation with respect to the argument). We measure the convergence rate of such an algorithm by the number of evaluations of the objective function required to attain the desired accuracy ε of the solution. In this paper, we focus on the order of growth in the number of evaluations of the objective function required to attain the prescribed accuracy ε of the solution as ε approaches zero. To design a reasonable algorithm, one should first define a class of functions that contains the objective functions of interest. The reason is that one cannot construct a fast method if the class of functions to be optimized is too broad. In this paper, fast optimization algorithms are obtained for "non-degenerate" objective functions. An optimization method is said to be fast if the number of evaluations of the objective function required for attaining the prescribed accuracy ε of the solution increases slowly (logarithmically) as ε tends to zero (that is, this number behaves as $O(|\ln \varepsilon|^u)$ for some $u > 0$). It is shown that the considered random search method is fast, and the corresponding number of evaluations of the objective function have the form $O(\ln^2 \varepsilon)$.

keywords: random search, global optimization, estimate of convergence rate.

6. Scheduling

Ilya Chernykh[1], **Olga Krivonogova**[1] **and Anna Shmyrina**[2]

Approximation algorithms for two-machine proportionate routing open shop on a tree

[1] Sobolev Institute of Mathematics, Russia
[2] Novosibirsk State University, Russia
idchern@math.nsc.ru, krivonogova.olga@gmail.com

The routing open shop problem is a natural generalization of the open shop problem and the metric traveler salesman problem. Jobs are located at the nodes of a transportation network, which has to be traversed by mobile machines in order to process the operations of the jobs, similar to the classic open shop environment. We consider the proportionate special case of this problem, in which for each job processing times of its operations are equal. This problem is known to be NP-hard even for the simplest case with two machines and two nodes. We present the tight optima localization interval for the two-machine problem with asymmetric transportation network being arbitrary tree, yielding a $\frac{7}{6}$ -approximation algorithm for this problem. Surprisingly, the same result holds for a more general problem where travel times are machine-dependent under the special condition, when one machine is "not faster" than the other. This stands in contrast to the general rout- ing open shop (without the proportionate condition), for which optima localization intervals are different for identical travel times and uniform travel times cases.

keywords: proportionate open shop, routing open shop, unrelated travel times, approximation algorithms, optima localization..

Yulia Zakharova[1]

Hybrid evolutionary algorithm with optimized operators for total weighted tardiness problem

[1] Sobolev Institute of Mathematics, Russia
julia.kovalenko.ya@yandex.ru

A new evolutionary algorithm with optimal recombination is proposed for the total weighted tardiness problem on the single machine. We solve the optimal recombination problem in a crossover operator. The NP-hardness of this problem is proved for various practically important cases. We construct the initial population means of greedy constructive heuristics. The insert and swap local search heuristics are used to improve the initial and the final populations. A computational experiment on the OR-Library instances shows that the proposed algorithm yields results competitive to those of well-known algorithms and confirms that the optimal recombination may be used successfully in evolutionary algorithms.

keywords: evolutionary algorithm, optimal recombination, scheduling, experiment.

Igor Vasilyev[1,2], **Ildar Muftahov**[2], **Anton V. Ushakov**[1]

MIP heuristics for a resource constrained project scheduling problem with workload stability constraints

[1] Matrosov Institute for System Dynamics and Control Theory, Russia
[2] Novosibirsk Research Center, Huawei Russian Research Institute, Russia
{vil,aushakov}@icc.ru

We address a variation of the multi-skill resource-constrained project scheduling problem motivated by the need to find a quality schedule for the construction activities related to rolling-out a large network of mobile stations. The network roll-out (NRO) is a complex industrial project aimed at deploying a set of mobile stations in certain region. This requires attracting a set of contractors, who can provide worker teams that have skills to execute specific project activities depending on their type. The problem also involves some additional specific industrial requirements related to how the contractors' workload should be distributed during the project duration (workload stability constraints). Other important industrial constraints are the precedence redundancy constraints on the project activities. We formulate this problem as a mixed-integer linear program that can be viewed as a disaggregated time-indexed integer programming formulation with pulse variables. To find quality feasible solutions of the problem, we propose a two-phase MIP heuristic consisting of (i) a relax-and-fix heuristic followed by (ii) a large neighborhood search. The effectiveness of the proposed solution approach is demonstrated in extensive computational experiments on real world problem instances.

keywords: multiple project scheduling, large neighborhood search, relax-and-fix heuristic, network roll-out, resource-constrained project scheduling, relax-and-optimize.

Alexander Lazarev[1]

Metric approach for scheduling problems

[1] V.A. Trapeznikov Institute of Control Sciences, Russia
jobmath@mail.ru

For any mathematical theory is very important to know the distance (metric) between objects. We consider metric in scheduling theory and how we can use for solution scheduling problems. Content of the metric approach we consider in details on the NP-hard in strong sense scheduling problem for single machine $1|r_j|L_{\max}$. All the polynomial solvable cases of the problem are limited by some system of linear inequalities. To find an approximate solution to the initial instance of the problem, we project the instance (the point in 3-n dimensional space) onto a solvable case of the problem, solving the linear programming problem. The distance between the initial point and the found point in the solvable case will be the upper bound of the absolute error of the objective function.

keywords: metric approach, scheduling theory, single-machine scheduling.

Aleksejs Lozkins[1], **Andrey Logachev**[1], **Ekaterina Gorynya**[1] **and Egor Berdin**

TEST CASES SCHEDULING ON THE TELECOMMUNICATION NETWORK

[1] Saint Petersburg State University, Russia
{a.n.logachev,berd_egor}@mail.ru

The scheduling problem arising in the test cases (TCs) execution on the telecommunication testbed is actual real-world task. The telecommunication equipment should be verified on the real or potential user scenarios before reaching the client. This study concentrates on the TCs scheduling problem on the network. The problem is formulated as mixed integer problem with objective function maximizing the testbed utilization rate. The amount of TCs in the task to be executed and wide possibility to assign TC on the network make the issue complex. To resolve the large scale problem we present column generation algorithm with heuristic batching of possible assignments of the TC to the testbed. The numerical experiments on the real data were performed using open source SCIP solver.

keywords: scheduling, column generation, networks, job shop problem.

Svetlana Malakh[1] **and Vladimir Servakh**[1]

PROJECT SCHEDULING PROBLEM WITH NPV CRITERION

[1] Sobolev Institute of Mathematics, Russia
svv_usa@rambler.ru

The paper considers the problem of maximizing the profit of an investment project with the possibility of using loans and reinvesting the income received. An approach to modeling is proposed in which labor, material and other resources can be transformed into a single type of resource – financial. The properties are investigated and algorithms for solving the problem are proposed.

keywords: project scheduling, investment, NPV, lending.

Vladimir Servakh[1]

THE COMPLEXITY OF THE PROBLEM OF MINIMIZING MAKESPAN FOR IDENTICAL JOBS

[1] Sobolev Institute of Mathematics, Russia
svv_usa@rambler.ru The problem of minimizing the total processing time of identical jobs with a complex technological route, when it is possible to

reentrant receive jobs on some machines, is considered. The paper investigates the computational complexity of this problem. Its NP-hardness in the usual sense is proved. Algorithms for constructing an approximate solution based on the use of cyclic schedules are proposed.

keywords: reentrant flow-shop, identical jobs, NP-hardness.

Sergey Sevastyanov[1]

EFFICIENT APPROXIMATION ALGORITHMS FOR SCHEDULING PROBLEMS WITH p-BATCH UNIFORM AND UNRELATED MACHINES AND ARBITRARY PROCESSING SET RESTRICTIONS

[1] Sobolev Institute of Mathematics, Russia
seva@math.nsc.ru

We consider the problem of computing an optimal schedule for the execution of n jobs on m parallel machines, in the framework of given processing set restrictions. The latter means that for each job j a set of admissible machines-executors is specified, and only one of them should be chosen for processing the job. Three types of machines (or three ways to define the duration of a job on a machine) are considered: (1) unrelated machines – a most general case, when for each pair (job, machine) a personal duration of the job on that machine is specified; (2) uniform machines – the case, when a volume of work (p_j) and a speed (s_i) are specified for each job j and each machine i, and the duration of job j is equal to p_j/s_i, when it is assigned to machine i; (3) identical machines – the case, when all machine speeds are equal to 1. The problems with these three types of machines are normally referred to as R-, Q-, and P-problem, respectively. We will call it an X-problem, when the type of machines is immaterial. Another feature of our problem is the possibility for some machines (so called batch machines) to perform the jobs not one by one, but in batches. All the jobs included in a batch are processed as if they are combined into one "job", the duration of which is calculated according to a given function of durations of the jobs in this batch. We consider the case when this function is the "maximum", which is equivalent to processing all jobs in a batch in parallel and is denoted as "p-batch" (or "parallel-batch"). Each machine processes its batches successively. The maximum number of jobs that could be included in each batch on a machine i is limited by a given integer B_i. This allows considering the mixed case, when there are both ordinary machines (with $B_i = 1$), and batch-machines (with $B_i > 1$). The objective is the minimum schedule length (or the minimum makespan). Our results. For X-problems in cases of $X \in \{P, Q, R\}$, we develop two efficient approximation algorithms for generating approximate solutions with a priori accuracy bounds: a "Short algorithm" with an absolute performance guarantee and a "Long algorithm" with a ratio performance guarantee. The operation of both algorithms differs when applied

to Q- and R-problems: while the running time of both algorithms applied to the R-problem is polynomial (and requires time needed for solving a linear program), for the Q-problem, we are able to design strongly polynomial algorithms. Thus, our algorithms outperform similar approximation algorithms known for this kind of problems from scheduling literature: our algorithms enable one to solve more general scheduling problems more efficiently and with better performance guarantees.

keywords: batch scheduling, parallel machines, approximation, polynomial-time algorithms.

Yulia Zakharova[1] **and Maria Sakhno**[1]

STRUCTURE OF SCHEDULES FOR PROBLEMS WITH PARALLELIZABLE JOBS

[1] Sobolev Institute of Mathematics, Russia

kovalenko@ofim.oscsbras.ru, sosnovskayamy@gmail.com

We consider scheduling problems with parallelizable jobs, arising in multiprocessor computer systems. Such jobs can be executed by more than one processor simultaneously. We investigate a block structure of the schedules for various series of instances. Procedures of local improvements are proposed and experimentally evaluated. Various lower bounds are provided and investigated in the context of the block structure of schedules. We also analyze cases with uncertain input data and resource constraints.

keywords: scheduling, algorithm, resource, parallel job.

The reported study was funded by RSF, grant no 22-71-10015.

7. Operations Research

Evgeny Bobrov[1,2] **and Adyan Dordzhiev**[3]

On probability shaping for 5G MIMO wireless channel with realistic LDPC codes

[1] Moscow Research Center, Huawei Technologies, Russia
[2] Lomonosov Moscow State University, Russia
eugenbobrov@ya.ru, adyandordzhiev@yandex.ru

Probability Shaping (PS) is a method to improve a Modulation and Coding Scheme (MCS) in order to increase reliability of data transmission. It is already implemented in some modern radio broadcasting and optic systems, but not yet in wireless communication systems. Here we adapt PS for the 5G wireless protocol, namely, for relatively small transport block size, strict complexity requirements and actual low-density parity-check codes (LDPC). We support our proposal by a numerical experiment results in Sionna simulator, showing 0.6 dB gain of PS based MCS versus commonly used MCS.

keywords: QAM, MCS, OFDM, 5G, PS, FEC, BICM, LDPC.

Chentsov A.G.[1,2] **and Chentsov P.A.**[1,2]

Additive routing problem for a system of high-priority tasks

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
[2] Ural Federal University, Russia
chentsov@imm.uran.ru, chentsov.p@mail.ru

It is considered the routing problem for which some fixed tasks must be serviced above all. Other tasks can be serviced only after realization of above-mentioned original tasks. It is supposed that each our task is the megalopolis (nonempty nite set) visiting with fullment of some works. In our setting, two partial interconnected routing problems arise. We suppose that, in each partial routing problem, the correspond- ing precedence conditions are given. Using widely understood dynamic programming (DP), we obtain the optimal composition solution for ini- tial total problem. As an application, we note the known engineering problem connected with sheet cutting by zones on CNC machines. By DP procedure the optimal algorithm realized on PC was constructed.

keywords: dynamic programming, precedence conditions, route.

Sergey Lavlinskii, Artem Panin and Alexander Plyasunov

Public-private partnership model with a consortium

[1] Sobolev Institute of Mathematics, Russia
chentsov@imm.uran.ru, chentsov.p@mail.ru

A model is proposed for generating a mineral raw materials development program in a resource-rich region. The model is based on a special mechanism

of public-private partnership with a consortium. The main idea of the partnership model is to cluster mine fields and set up a system of consortia of private investors who jointly implement projects to construct the necessary production infrastructure in the cluster. Such a mechanism is based on the search for a compromise between the interests of the government and private investors, ensuring a Stackelberg equilibrium. In the process of interaction (two periods, sequential choice), the government acts as a leader by setting quotas on the compensations for the consortia's costs of implementing the infrastructure projects. The system of consortia plays the role of a follower by rationally choosing the infrastructure development program that ensures the profitability of the development projects for private investors, taking into account the costs of shared construction and the compensation schedule offered by the government. This approach allows one to form a targeted development plan by solving a bilevel problem of mathematical programming. This plan determines for each consortium a list of implemented infrastructure projects and, for private investors, a schedule of infrastructure costs and their compensations from the budget. It is proven that the problem of the government belongs to the class of $\sum_2^P$-hard problems associated with the second level of the polynomial hierarchy. The main directions are proposed in searching for efficient solution algorithms based on metaheuristics and enabling the solution of high-dimensional problems.

keywords: Stackelberg game, bilevel mathematical programming problems, $\sum_2^P$-hard problems, stochastic local search, strategic planning, public-private partnership, a consortium of subsoil users.

Tatiana Levanova[1], **Alexander Gnusarev**[1], **Ekaterina Rubtsova** [2] **and Vyatcheslav Sigaev**[3]

VARIABLE NEIGHBORHOOD SEARCH APPROACH FOR THE BI-CRITERIA COMPETITIVE LOCATION AND DESIGN PROBLEM WITH ELASTIC DEMAND

[1] Sobolev Institute of Mathematics, Russia
[2] Dostoevsky Omsk State University, Russia
[3] Avtomatika-Servis LLC, Russia
chentsov@imm.uran.ru, chentsov.p@mail.ru

In this paper, we develop a bi-criteria approach to solving the competitive location and design problem with elastic demand. The problem involves a new company, its competitor, and its consumers. The competitor has already placed its enterprises. The new company can choose the locations and the design variants for its facilities within the budget. Consumers independently choose service points from the open facilities of the company or competitor based on their preferences. The goal of the new company is to capture the largest possible share of the total demand. This situation is described using

a non-linear integer programming model. In real situations, demand data and other parameters may change. In this case, it is necessary to make a decision that would be stable regarding such changes. We consider one of the concepts of robustness and formulate a new bi-criteria statement of the problem under consideration. In addition to the criterion that maximizes the share of the total demand, it also contains a criterion that maximizes the robustness of the solutions obtained. To solve the bi-criteria problem, we propose an algorithm based on variable neighborhood search and a modified version of the SEMO evolutionary algorithm. The features of the problem and the presence of two criteria are taken into account. Experimental studies have been carried out. The quality of the solutions obtained is analyzed, and a comparison with previous developments is discussed.

keywords: robustness, bi-criteria optimization, integer programming, competitive location problem, variable neighborhood search.

Alexander Yuskov[1], **Igor Kulachenko**[2], **Andrey Melnikov**[2] **and Yury Kochetov**[2]

DECOMPOSITION APPROACH FOR SIMULATION-BASED OPTIMIZATION OF INVENTORY MANAGEMENT

[1] Novosibirsk State University, Russia

[2] Sobolev Institute of Mathematics, Russia

a.yuskov@g.nsu.ru, {ink,melnikov,jkochet}@math.nsc.ru

We consider a two-echelon inventory management problem, where customers' requests for spare parts of different types must be fulfilled within a given service level threshold. The supply system is composed of multiple warehouses in the first echelon, where the customers' requests are processed, and a single second-echelon warehouse, replenishing stocks of the first-echelon warehouses. Replenishment requests of warehouses are invoked according to inventory policies, which are characterized by one or two numerical parameters and are individual for each warehouse and each spare part type. The goal is to minimize the total storage cost for all warehouses at both echelons. System operation is simulated within a black-box function that computes the request satisfaction rate and inventory holding costs depending on the policy parameters. In the work, we propose a decomposition approach to adjust these parameters for an industrial-sized supply system. Computational experiments for up to 1,000 types of items and 100 warehouses are discussed.

keywords: grey-box optimization, multiple-choice knapsack problem, local search.

Mostafa Abotaleb[1], Amr Badr[2], Tatiana Makarovskikh[1] and Anatoly Panyukov[1]

IMPROVING SMART FARMING BY OPTIMIZING TRANSFER LEARNING FOR WEED DETECTION

[1] South Ural State University, Russia

[2] University of New England, Australia

paniukovav@susu.ru, amr.mostafa@live.com

Over the past few years, weeds have been to blame for losses in agriculture. To solve this issue, farmers must evenly spray weedicides across the entire field, which uses a lot of weedicides, on the other hand the use of weedicides has an impact on the environment. The detection of weeds is currently a concern in field robotics, where it is vital to limit the use of phytochemical products on crops. Weed detection in dense cultures is a plant science problem. The detection of weeds in crops will aid in increasing crop production while reducing crop production losses. This paper presents a method of weed detection for the control and prevention of particular weeds. The data is separated into training, validation, and testing datasets by gathering the plants and weeds datasets, which are grayscale photos. The data is then sent to the convolutional neural network. The model can identify weeds among plants based on the knowledge it has acquired. The application of transfer learning for a pretrained model RCNN to optimize the hyperparameter for weed detection in dense cultures demonstrated improved performance in comparison to the state of the art without the requirement of large data or high computational power for training. This was accomplished by optimizing the hyperparameter.

keywords: transfer learning, optimization, RCNN, weed detection, object detection, convolutional neural networks, computer vision.

Farid Ahmatshin[1], **Irina Petrova**[1] **and Lev Kazakovtsev**[1,2]

JAMES-STEIN SHRINKAGE IN THE PROBLEM OF AUTOMATIC GROUPING OF INDUSTRIAL PRODUCTS BY HOMOGENEOUS PRODUCTION BATCHES

[1] Reshetnev Siberian State University of Science and Technology, Russia
[2] Siberian Federal University, Russia
emailahmatshin_fg@mail.sibsau.ru

It is known that the use of shrinkage can significantly improve the accuracy of K-means cluster analysis for a relatively wide range of data. In this article we ask about the effectiveness of the applicability of James-Stein shrinkage estimation in the problem of automatic grouping of industrial products into homogeneous production batches. We compare the shrinkage results with the traditional k-means algorithm. The data we used in this study are taken from test results performed at the test center for batches of integrated circuits. The dataset was normalized by the values of permissible drift, normalized by acceptable parameter, and normalized by standard deviation. Using the Rand index we demonstrate that the accuracy increases significantly in the simulated data in the problem of automatically grouping industrial products into homogeneous production batches. The use of an improved James–Stein shrinkage estimate reduces the influence of uninformative parameters of normalized to acceptable parameter values.

keywords: k-means, clustering, greedy Heuristics, electronic radio products, Stein estimation.

Dmitry Arkhipov[1], **Daniel Skachkov**[1], **Dong Zhang**[2] **and Jie Ren**[3]

COMPUTATIONAL TREE-BASED TOOL TO SOLVE INDUSTRIAL OPTIMIZATION PROBLEMS

[1] Moscow Institute of Physics and Technology, Russia
[2] Algorithm & Technology Development Department of Huawei, China
[3] Huawei Global Technical Service, China
miptrafter@gmail.com

Industrial optimization problems are often large-scaled and have many business requirements. Problem formulation may change multiple times during iterative process of clarification it statement with business. We faced the situation when MILP solvers are not able to find feasible solution in suitable time and usage of commercial heuristic-based solvers (e.g. Local Solver https://www.localsolver.com/) is limited. We propose a tool to solve Mathematical Programming problems which brings together several ideas to make it efficient and flexible to problem statement changes. Tool builds a computational tree (CT) - a data structure was introduced by Laurent Michel & Pascal Van Hentenryck in 2000 and plays a key role in Local Solver. CT allows to quickly recalculate objectives and constraints after changing values of prob-

lem variables. Tool allows user to define variables with attributes. Variable attributes is being used to guide the search process. We present a method of defining neighborhood for thermodynamic simulated annealing (Juan de Vicente, et. al. 2003). We also present a data structure to support the choice of local steps in a constant time. The developed tool is applied to Vector Bin Packing Problem and two industrial problems stated as MP. Numerical experiments and comparison analysis with MILP solvers and open-source OptaPlanner solver (`https://www.optaplanner.org/`) are presented.

keywords: MILP solvers, mathematical programming, computational tree (CT), simulated annealing, open-source OptaPlanner solver.

Pavel Borisovsky[1]

PARALLEL EVOLUTIONARY ALGORITHM FOR THE RECONFIGURABLE TRANSFER LINE BALANCING PROBLEM

[1] Sobolev Institute of Mathematics, Russia
`pborisovsky@ofim.oscsbras.ru`

This paper deals with an industrial problem of machining line design, which consists in partitioning the given set of operations into several subsets corresponding to workstations and sequencing the operations to satisfy the technical requirements and achieve the best performance of the line. The problem has a complex set of constraints that include partial order on operations, part positioning, inclusion, exclusion, cycle time and installation of parallel machines on a workstation. The problem is NP-hard and even finding a feasible solution can be a difficult task from the practical point of view. A parallel evolutionary algorithm is proposed and implemented for execution on a Graphics Processing Unit (GPU). The parallelization in the EA is done by working on several parents in one iteration and in multiple application of mutation operator to the same parent to produce the best offspring. The proposed approach is evaluated on large scale instances and demonstrated superior performance compared to the algorithms from the literature in terms of running time and ability to obtain feasible solutions.

keywords: line balancing, CNC machines, partial order, setup times, split decoder, parallel computing, GPU.

Anton Eremeev[1] **and Dmitry Silaev**[2]

On hardness of Funnel benchmarks family

[1] Sobolev Institute of Mathematics, Russia
[2] Dostoevsky Omsk State University, Russia
eremeev@ofim.oscsbras.ru, chuvakeu2001@gmail.com

The paper presents theoretical analysis of performance of non-elitist evolutionary algorithms (EAs), when they are applied to a family of fitness functions Funnel, introduced by Dang et al (2021). Using a simplified level-based analysis, we obtain a new polynomial upper bound on the expected runtime of the EA with a recently proposed power-law selection on Funnel.

keywords: Evolutionary algorithm, power-law selection, runtime, benchmark.

Sergey Gladyshev and Dmitry Arkhipov

Resource capacity bounding algorithms for Resource-Constrained Project Scheduling

[1] Moscow Institute of Physics and Technology, Russia
gladyshev.si@phystech.edu, miptrafter@gmail.com

The generalization of Resource Constrained Project Scheduling Problem (RCPSP) with piecewise-constant resource capacities is considered. The problem consists of scheduling activities with respect to the precedence and resource constraints. RCPSP is a well-studied NP-complete problem, widely used in practice, which stimulates its studies in area of Management Science and Operations Research. One of the most popular and effective approach to solve RCPSP is Constraint Programming. In this research we propose new constraint propagation algorithms, which tightens the feasible region by exploring resource usages. To evaluate the efficiency of proposed approaches two series of numerical experiments using PSPLIB benchmark (https://www.om-db.wi.tum.de/psplib/) were made. We prove that resource capacity bounding improves performance of state-of-the-art RCPSP constraint propagators by comparing domain decreasing with and without proposed propagators and using proposed algorithms on pre-solve stage and evaluating the speed up of MILP and CP solvers.

keywords: RCPSP, constraint programming, project scheduling, project management, scheduling.

Sergey Golovanov[1,2], **Guzel Shkaberina**[1], **Vladimir Kazakovtsev**[1,2] **and Lev Kazakovtsev**[1,2]

Homogeneity assessment of a set of objects on the basis of the quality characteristics of the division of a set into groups

[1] Reshetnev Siberian State University of Science and Technology, Russia
gsm-itc@yandex.ru, z_guzel@mail.ru, levk@bk.ru

The paper describes an approach to determining the homogeneity of a set of objects based on the characteristics of the quality of dividing a set into groups. We have formulated the concept (optimization problem model) of homogeneity based on the introduced characteristics and proposed a general approach to determining the area of qualitative division using training samples. We also gave an example of the practical use of the described approach to determine the homogeneity of batches of electronic components in the process of additional tests for space applications.

keywords: clustering, clustering quality, electronic components quality, homogeneity.

Alexander Gornov[1], **Anton Anikin**[1], **Pavel Sorokovikov**[1] **and Tatiana Zarodnyuk**[1]

Numerical study of large-scale atomic-molecular clusters

[1] Matrosov Institute for System Dynamics and Control Theory, Russia
gornov.a.yu@gmail.com, anton.anikin@gmail.com

The paper proposes an approach focused on the search for optimal structures of atomic-molecular clusters of large-scale dimensions by minimizing low-potential functions of a given structure. Using the developed computational techniques, a numerical study of the Keating (up to 10^8 atoms), Morse (up to 1100 atoms), Gupta (up to 1100 atoms), Sutton-Chen (up to 150 atoms) and Z1 (up to 108 atoms) atomic-molecular clusters was carried out. A comparative analysis of the experimental results did not reveal any sharp deviations from the observed regularity of the found potential values, which describes their growth depending on the number of atoms. The authors are not aware of other attempts to solve these problems for considerably large dimensions.

keywords: atomic-molecular clusters, minimizing low-potential functions.

Ivan Davydov[1] **and Alexander Lyapin**[2]

MATHEURISTIC WITH VLNS APPROACH FOR ADDITIONAL ENGLISH COURSES TIMETABLING

[1] Sobolev Institute of Mathematics, Russia
[2] Novosibirsk State University, Russia
i.a.davydov@math.nsc.ru

We consider a timetabling problem that arises in a school, which provides an additional educational service, improving English language knowledge. During the summer break the school collects the applications from potential students. Applicants indicates the set of time windows they are able to attend the lessons in and take a part in a trial lesson to define their level of knowledge. Then the school runs the schedule construction process. Students can be united into a group if they have similar age, the same level of knowledge, and shareable time windows. The size of a group is limited. Students who do not fit the current schedule, are kept on the waiting list until some more applications arrive. The process then is repeated, additional groups and lessons are added to the previous schedule. The objective of the school is two-fold: maximize the number of students attending while keeping a high number of students in a group on average. Wwe provide a mathematical model for the described problem in terms of integer linear programming and propose a constructive heuristic to solve the entire problem combined with a MIP solver to improve the obtained solution further. Computational results carried on real and artificially generated instances shows the efficiency of the proposed approach.

keywords: timetabling, matheuristic.

Daniil Khachai[1], **Olga Battaïa**[1], **Alexander Petunin**[2] **and Michael Khachay**[3]

CUTTING PATH PROBLEM: GENERAL MODEL AND ALGORITHMS

[1] KEDGE Business school, France
[2] Ural Federal University, Russia
[3] Krasovsky Institute of Mathematics and Mechanics, Russia
{daniil.khachay,olga.battaia}@kedgebs.com,
aapetunin@gmail.com, mkhachay@imm.uran.ru

We propose a general approach to modeling discrete version of the well-known industrial Cutting Path Problem (CPP) and a algorithmic framework allowing to find its close-to-optimal and optimal solutions. The approach relies on reducing the CPP to the Precedence Constrained Generalized Traveling Salesman Problem and the recent algorithmic results obtained by the authors for the latter problem

keywords: Cutting Path Problem, Generalized Traveling Salesman Problem, precedence constraints, Adaptive Large Neighborhood Search, branch-and-cut.

Oleg Khamisov[1] **and Nadezhda Ulyanova**[1]

POWER GENERATION PLANNING IN ENERGY SYSTEMS

[1] Melentiev Energy Systems Institute, Russia
globopt@mail.ru

We consider the problem of finding the optimal functioning of a set of energy units, which provides the necessary total load at each time interval. It is required to minimize the total costs incurred by the units to provide the necessary load. The costs of each unit include variable costs as a quadratic function of the amount of energy produced by the unit and constant costs. The model incorporates the periods of unit startup and unit shutdown, in which the units incur costs but do not produce energy. Using binary variables that determine the state of units, we formulate the problem under consideration as a mixed integer quadratic programming problem. The solution is based on approximations of the problem by mixed integer linear programming problems obtained via replacing the quadratic function of variable costs by its piecewise linear approximations. An iterative algorithmic scheme that uses these approximations was proposed and its numerical realization in the AIMMS environment was carried out. We validated the performance of the scheme on test examples and compared it with known solvers.

keywords: energy system, optimization, mixed integer quadratic programming, linear approximations, numerical algorithm.

Oleg Khamisov[1]

BANG-BANG MIXED SPEED GOVERNING FOR STEAM AND HYDRO TURBINES

[1] Melentiev Energy Systems Institute, Russia
globopt@mail.ru

Any electrical gird continuously goes through changes in electrical load. In generating units this causes a mismatch between electrical and mechanical torques resulting in deviations of turbine speed. At the same time turbines can operate only in very narrow range of speeds; thus, a real-time control is applied to turbine input valves. Normally the control is given by proportional integral (PI) controller with gains set by grid standards. At the same time, the optimal control problem, can be formulated as minimization of speed deviation integral under a set of differential equations modelling the system. It can be shown that the solution should be in a form of bang-bang control. Derivation of such control is complicated due to general non-observability of turbine model. Therefore, in this paper we derive a mixed control system. It acts as an optimal bang-bang control at the parts of system trajectory, where speed deviations are large enough for obtaining optimal control despite non-observability.

For the rest of the trajectory, control acts similar to the PI control. The theoretical results are supported by numerical experiments with detailed high-order models of steam and hydro turbines connected to synchronous generator.

keywords: proportional-integral control, bang-bang control, power systems.

Ivan Khmara[1] **and Tatiana Levanova**[1]

DEVELOPMENT OF LOCAL SEARCH ALGORITHMS FOR THE ROBUST P-MEDIAN PROBLEM

[1] Sobolev Institute of Mathenatics, Russia
{ivan-hmara, levanova}@mail.ru

In this paper, we develop algorithms for finding a solution to the robust p-median problem. In the classical formulation of the p-median, a set of facilities and a set of customers are given. The costs of servicing each client from each facility and the volume of demand for each client are known. It is necessary to open exactly p facilities and serve customers with them so that the costs are minimal. In conditions of market volatility, the parameters of the problem may change. Often it turns out that customer demand is unstable. Then there is a need to optimize some stability measure. Robust optimization is one of the approaches to solving this problem. In this paper, we consider a robust version of the p-median problem, where stability associated with customer demand is optimized. To solve this problem, local search algorithms are proposed. Experimental studies of the proposed algorithms are carried out, their results are analyzed.

keywords: discrete location, p-median problem, robustness, local search.

Vitalii Kochevadov[1] **and Artem Sedakov**[1]

DYNAMIC NETWORK MODEL OF PRODUCTION AND INVESTMENT

[1] Saint Petersburg State University, Russia
a.sedakov@spbu.ru

The paper studies a dynamic network game that models the competitive behavior of firms in a market. It is assumed that firms, under the condition of simultaneous and independent choice of their actions, implement the behavior that determines their production and investment behavior at each point in time. The production behavior of the firm reflects the ongoing quantities that it should produce and supply to the market. The investment behavior specifies the ongoing amounts of investment that the firm allocates to the modernization of its production technology in order to prevent it from becoming obsolete. Next, the unit cost is assumed to depend on the firm's investment and the investment of its competitors, which are determined by an exogenous network.

Two types of Nash equilibria are characterized: open-loop and feedback. Finally, we analyze the impact of the network and related model parameters on firms' behavior, profits, and competitive advantage.

keywords: competition, investment, dynamic game, network, Nash equilibrium.

This research was supported by the RSF, grant no. 22-11-00051, https://rscf.ru/en/project/22-11-00051/

Aleksei Kondratev[1], **Egor Ianovski**[1] **and Alexander Nesterov**[1]

GEOMETRIC AND OPTIMAL SCORING RULES IN SPORTS COMPETITIONS

[1] HSE University, Russia

kondratev.aleksei@yandex.ru, george.ianovski@gmail.com, asnesterov@hse.ru

Scoring rules are widely used to rank athletes in sports. Each position in each individual ranking is worth a certain number of points; the total sum of points determines the aggregate ranking. The question is how to choose a scoring rule for a specific application. First, we derive a one-parameter family with geometric scores which satisfies two principles of independence: once an extremely strong or weak athlete is removed, the aggregate ranking ought to remain intact. This family includes Borda count, generalised plurality (medal count), and generalised antiplurality (threshold rule) as edge cases, and we find which additional axioms characterise these rules. Second, we introduce a one-parameter family with optimal scores: the athletes should be ranked according to their expected overall quality. Finally, using historical data from biathlon, golf, and athletics we demonstrate how the geometric and optimal scores can simplify the selection of suitable scoring rules, show that these scores closely resemble the actual scores used by the organisers, and provide an explanation for empirical phenomena observed in biathlon and golf tournaments. We see that geometric scores approximate the optimal scores well in events where the distribution of athletes' performances is roughly uniform.

keywords: OR in sports, rank aggregation, Borda count, sports ranking, ranking system.

Nikita Kosyanov, **Elena Gubar** **and Vladislav Taynitskiy**

MPC CONTOLLERS IN SIIR EPIDEMIC MODEL

[1] Saint Petersburg State University, Russia

v.taynitsky@spbu.ru

Infectious diseases are one of the most important problems of the modern world, e.g. the periodic outbreaks of coronavirus infections caused by COVID-19, influenza and many other respiratory diseases have significantly affected the

economies of many countries (lockdowns, masks and reduced production efficiency). It is therefore important to minimize the economic damage, which includes both loss of work and treatment costs. Recent studies have presented many different models describing the dynamics of virus spread, which help to analyze and explain epidemic outbreaks. Previously, it was shown that several infections can circulate in the population at the same time. In this case, the controlled Susceptible Infected Recovery model with multiple infected subgroups can be used. The solution of this model is usually found by using classical optimality criteria, e.g. the Pontryagin maximum principle. In the current study we focus on finding solutions that are robust to noise and take into account the dynamics of future changes in the process. We extend previous results by using a nonlinear model-predictive-control (MPC) controller to find effective controls. MPC is a computational mathematical method used in dynamically controlled systems with observations to find effective controls. The main applications of this method are in the petrochemical, woodworking and energy industries, where, among other things, the stability of the obtained controllers and the possibility of their automatic adjustment are important. The MPC controller finds a solution in the form of piecemeal functions on the control interval, but the optimization problem is solved over a larger time interval — the forecast horizon. The MPC controller finds the solution in the form of piecewise constant functions on the control interval, but the optimization problem is solved on the forecast horizon. As the main contribution of this work, we formulated a control problem and solved it using an MPC controller, a series of numerical experiments are carried out to confirm the effectiveness of the solutions obtained by the MPC controller for the SIIR model.

keywords: epidemic processes, SIIR models, optimal control, predictive control, linearization of dynamical systems.

Alexander Krylatov[1,2] **and Anastasiya Raevskaya**[1]

HUB OPERATION PRICING IN THE INTERMODAL TRANSPORTATION NETWORK

[1] Saint Petersburg State University, Russia
[2] Solomenko Institute of Transport Problems, Russia
a.krylatov@spbu.ru

The growing importance of strategic supply chain management in light of intermodal transportation networks has attracted increased attention from researchers in recent decades. The study of intermodal logistic services has become urgent for multiple branches of science. This work is focused on hub operation pricing under an equilibrium freight flow assignment model. We formulate this model as a non-linear optimization problem and show that its solution corresponds to the equilibrium assignment pattern in an intermodal transportation network. We analyze the sensitivity of the average purchase cost

and hub load to different strategies of hub operation pricing. To this end, we obtain the equilibrium freight flow assignment pattern in an explicit form for the network with one consumer-supplier pair, a single layer of hubs, and affine performance functions. The findings of the paper can give fresh managerial insights; in particular, we show risks arising within available pricing strategies.

keywords: nonlinear optimization, freight flow assignment, intermodal transportation.

The work was supported by the RSF, grant no. 22-71-10063

Anna Lempert[1], **Alexander Kazakov**[1] **and Lebedev Pavel**[2]

Algorithms of optimal routing and covering for changing environment

[1] Matrosov Institute for System Dynamics and Control Theory, Russia

[2] Krasovsky Institute of Mathematics and Mechanics, Russia

{lempert, kazakov}@icc.ru, pavel_l_@mail.ru

The report deals with the problems of constructing optimal routings between several points and the thinnest coverings for a convex set by a set of similar elements. As a distance between two points, we use the shortest time it takes to achieve one point from another, and the boundary of each covering circle is an isochron. Such problems arise in applications, particularly in sonar and underwater surveillance systems. To solve the problems of covering with such circles and balls, we previously proposed algorithms based on the union of variational principles and geometric methods. The purpose of this study is to construct coverings when the characteristics of the environment change over time. We propose computational algorithms based on the theory of wavefronts and prove the statement about its properties. Illustrative calculations are performed.

keywords: optimal routing, optimal covering, wave front, dynamical metric, Chebyshev center.

Sergey Loktev[2], **Evgeny Bobrov**[1] **and Zongdi Yue**[2]

Geometric shaping for wireless communications

[1] Lomonosov Moscow State University, Russia

[2] Moscow Research Center, Huawei Technologies, Russia

s.loktev@gmail.com, yuezongdi@huawei.com

Geometric shaping is an optimization problem about modulation of a discrete signal by electromagnetic waves in a noisy environment. In this talk, we discuss information theory approach to this problem. This approach is already implemented in ATSC 3.0 private TV network. The goal of this talk is to reveal possible applications to wireless telecommunication. We discuss implementation of multidimensional geometric shaping. In particular, a method usually known as Probabilistic shaping is indeed a particular case of multidimensional

geometric shaping. We show that the target function for geometric shaping is maximal in the minimum of binary cross-entropy that allows usage of neural networks for maximizing it.

keywords: wireless, MCS, constellation, geometric shaping, labeling, FEC, coderate, BLER, AMC, MU-MIMO, SE, GMI, BICM, mutual information, QAM, probabilistic shaping.

Elena Lubnina and Yury Kochetov

THE VNS ALGORITHM FOR PEOPLE TRANSPORTATION PROBLEM IN OIL AND GAS OFFSHORE PROJECTS

[1] Sobolev Institute of Mathematics, Russia

jkochet@math.nsc.ru

High expenses of producing oil and gas in offshore field development projects implies oil companies to decrease operational costs, including expenses connected with the cargo and personnel transportation. In this paper, we consider the problem of people transportation in a planned time horizon. We need to deliver workers at the offshore platforms every day and return other workers to depot, due to crew changes, to do repair work or well tests, managements visits or other. We have a heterogeneous fleet of helicopters with known capacity. Moreover, we consider the possibility of delivering personnel by other sea transport. We can split the demand for each platform and visit it by different transport. Our goal is to find the routes to satisfy all demands with minimal total travel length. To tackle this pick-up and delivery routing problem, we design the MILP model and adopt the VNS-algorithm. Some hard constraints are relaxed and included into the objective function with penalties. Computational results for the real world test instances of the JV Vietsovpetro company are discussed.

keywords: periodic VRP, split delivery and pick-up, local search, penalty function, heterogeneous fleet.

Aleksandr Maslovskiy, Alexander Gasnikov[1,2,3], Denis Vorkutov[4], Oleg Sumenkov[4] and Ivan Tarabukin[4]

ALTERNATIVE MODEL DESIGN TECHNIQUE FOR DIGITAL PREDISTORTION

[1] Moscow Institute of Physics and Technology, Russia

[2] Kharkevich Institute for Information Transmission Problems, Russia

[3] Caucasus Mathematical Center of Adygh State University, Russia

[4] Sirius University, Russia

gasnikov.av@mipt.ru

In this paper, we investigate different alternative ideas for the design of digital pre-distortion (DPD) models for radiofrequency power amplifiers (RF-PA). When compared to the greedy search algorithm, these algorithms identify the model parameter combinations faster while still performing reasonably well.

For the subsequent implementation, different metrics of model costs and score results in the process of optimization enable us to achieve sparse selections of the model, that balance the model accuracy and model resources (according to the complexity of implementation). The results achieved in the process of simulations show that combinations obtained with explored algorithms show the best performance after a lower number of simulations.

keywords: digital pre-distortion, radiofrequency power amplifiers, TPE, NSGAII, QMC, CMA-EA.

Vladimir D. Mazurov[1] **and Anna V. Sharf**[1]

THE FORMALIZATION AND AN INFORMALIZABLE

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
vldmazurov@gmail.com, sharf.anna@yandex.ru

We propose a general framework to make insufficiently formalized problem more formal. Our framework is based on the classic Hahn and Banach Theorem, Novikoff's Theorem, Cassirer's philosophy of science, and our recent results.

keywords: informal problems, Hahn and Banach Theorem, Novikoff's Theorem.

Andrey Melnikov[1] **and Vladimir Beresnev**[1]

UPPER BOUND FOR COMPETITIVE FACILITY LOCATION PROBLEM WITH MULTI-STAGE FOLLOWER'S DECISION

[1] Sobolev Institute of Mathematics, Russia
{melnikov, beresnev}@math.nsc.ru

We consider a Leader-Follower competitive facility location model, where competing parties make decisions that consider set of customers' changes, happening on a planning interval of a predefined length. Leader makes their decision in the very first time period, while Follower is enabled to revise their decision in each time period of the interval. In the work, we study perspectives to develop an exact method for the Leader's problem. The method is based on using a high-point relaxation of a bi-level mathematical programming model, formalizing the problem, to compute upper bounds. The key element of the approach is a procedure to generate additional constraints strengthening the HP-relaxation. New specific constraints, supplementing c-cuts introduced in our previous works, are suggested to improve the upper bound quality.

keywords: Stackelberg game, multi-stage model, bi-level optimization, location.

Anton Mikhailov[1] **and Sergey Kumkov**[1]

GUIDING AIRCRAFT TO CHECKPOINT BY AIR-TRAFFIC CONTROLLER AND AUTOPILOT TOGETHER

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
tigr-mav@yandex.ru, sskumk@gmail.com

One of the traditional problems in the air-traffic management is to work out an optimal safe queue for merging aircraft flows at a point of a air-routes join. This problem implies another one, which is studied much less in the existing literature. It is the problem of guiding an aircraft to a prescribed point of its trajectory at some given instant. So, one should construct a trajectory of the aircraft (along known elements of the air zone scheme) and a velocity regime of its motion. Such a control could be produced both automatically by the autopilot and manually by an air-traffic controller (ATC) according to the actual situation in the air-traffic zone. An optimal solving this problem by a controller is difficult, especially, during rush hours when there are a lot of aircraft in the air. In this work, an optimizational formalization of this problem is suggested in the framework of mixed integer quadratic programming (MIQP). The criterion to be minimized is the number of ATC-pilot interactions necessary to provide the desired motion of the aircraft. Earlier, the authors created a formalization of this problem without taking into account the possibility of autopilot control. The obtained problem is solved with the help of a MIQP-solver from the Gurobi library, version 10.0. Possible trajectories of the aircraft in the air-traffic zone scheme are enumerated by a depth-first search. The optimal velocity regime along each trajectory is generated in 0.01-0.5 seconds. In general, procession of one aircraft takes about 1-5 seconds. This is acceptable for implementation of this procedure in real-time applications.

keywords: air-traffic management, prescribed arrival instant, trajectory and velocity regime generation, controller and autopilot control, mixed integer quadratic programming.

Natalia Nikitina[1] **and Evgeny Ivashko**[1,2]

CENTRALITY ANALYSIS OF THE ROAD NETWORK OF PETROZAVODSK

[1] Institute of Applied Mathematical Research, Karelian Research Center, Russia
[2] Petrozavodsk State University, Russia
{nikitina, ivashko}@krc.karelia.ru

Centrality characterizes the extent to which a vertex of the graph is close to its structural "center". Such a characteristic is a real-valued function on the set of vertices and expresses the potential importance of the vertex in terms of the structural properties of the graph, allowing the ranking of the vertices. Centrality is a powerful tool for graph analysis. In this paper we use centrality measures to analyse the road network of Petrozavodsk. We describe

the methods used to construct and enrich the road graph, our approach to centrality-based analysis and the results of numerical experiments. The results show that centrality analysis can be useful for roads structure analysis and planning of the roads structure development.

keywords: centrality, road structure, road graph, graph analysis, betweenness centrality.

Artem Panin, Alexander Plyasunov and Maksim Vodyan

A study of the threshold stability of the bilevel problem of facility location and discriminatory pricing

[1] Novosibirsk State University, Russia

apljas@math.nsc.ru

This work considers the problem of threshold stability in the problem of location and pricing. The initial problem is formulated as a Stackelberg game "leader - customers", in which the leader places facilites producing homogeneous products and sets prices for each of the open facilites using a discriminatory pricing strategy. Each customer has a budget and a unit demand. They choose the facility at which the minimum of their total costs (transportation and product purchase costs) is achieved and make a purchase if their costs do not exceed their budget. The goal is to maximize the leader's revenue. In the threshold stability problem, it is necessary to select a feasible solution for the location and pricing problem and such a maximum deviation from the current customer budgets, at which the leader's revenue will not be less than a predetermined value (threshold). In this work, two approximate algorithms for solving the threshold stability problem are developed, based on the VND heuristic. The idea of these algorithms is to find an approximate value of the radius of threshold stability based on the optimal solution of the initial problem with a fixed location of facilites. The algorithms differ in the way different locations of facilites are compared, which ultimately leads to different estimates of the radius of threshold stability. The numerical experiment showed the effectiveness of the chosen approach to constructing approximate solutions for the investigated problem, both in terms of the algorithms' runtime and the quality of the obtained solutions.

keywords: threshold stability, bilevel problem, facility location, discriminatory pricing.

The work was supported by the RSF, grant no 23-21-00424.

Liudmila Prokudina and Mikhail Vikhirev

Computer simulation of optimal flow regimes of thin layers of viscous liquid in film apparatuses

[1] South Ural State University, Russia

prokudinala@susu.ru, vikhirevmp@susu.ru

This work presents results of computational modeling of ver- tical liquid film

flow at moderate Reynolds numbers in the framework of partial differential equation of the state of the free surface of the liquid film. Equation coefficients include different physicochemical factors such as surface tension parameter. Algorithms of computation of wave characteristics of liquid film flow and free surface state of a viscous fluid thin layer are developed in Python. Computational experiments were carried out for the vertical water and alcohol films. Wave characteristics (frequency, increment, phase velocity) of liquid film flow are calculated. Optimal flow regimes of the liquid film characterized by the maximum increment and the minimum phase velocity are revealed. Comparison of water and alcohol film flow shows that alcohol film flow is more unstable: it is characterized by expansion of instability region width. Results could be used in the design or modernization of existing equipment and in the design of technological processes in liquid films.

keywords: liquid film, surface tension parameter, instability, moderate Reynolds numbers.

Varvara Rasskazova[1]

ILP model in solving RCPSP at the flow type production

[1] Moscow Aviation Institute, Russia
varvara.rasskazova@mail.ru

The paper is devoted to investigation of the problem regarding optimal planning at the steelmaking converter shop-floor. There proposed an integer linear program, which takes into account all significant technological restrictions. An objective is minimizing of total cost for the daily task processing, which leads to significant decreasing of the cost for production in general. Numerical experiments were carried out using real-world data and show a powerful and effectiveness of the proposed approach.

keywords: combinatorial optimization, integer linear programming (ILP), scheduling theory.

Varvara Rasskazova[1] **and Dmitriy Berenov**[1]

Object relation technique for modelling of digital production solutions

[1] Ural Federal University, Russia
varvara.rasskazova@mail.ru

A field of industry 4.0 is one of the most perspective with respect to research and optimisation of production and industrial problems. The main problems in this field are connected with necessary analysis and processing of a huge data, which have a different native and sources. Particularly, the data from various automatic sources, as well as getting due to human management, become far away from each other both from the structure point of view and the native

meaning. And there often occur, that such a different data in integrated form could contain a significant information for analysis, and moreover for a transformation of the technological process. At the same time, it is rather difficult to provide an analysis in frame of a significant asynchrony, and sometimes it becomes to a practical unsolveble problem. In this regard, the problem on developing of specialized methods for collecting, storing and processing big data of various structures are becoming fundamentally relevant. This is the purpose of the present work, which proposes a new method for organizing data based on the concept of object relations as a universal structure for production modeling. The proposed method is based on the concept of an "object", which is any entity of the production chain (plant, workshop, machine, production operation, unit of production, etc.) The key difference between the method of object relations and agent modeling is the fact that no behavioral scenarios are imposed on objects. Thus, the 'object' structure turns out to be authorized to enter into any relationship with other "objects" within the framework of the model under consideration, which in turn allows to accumulate various kinds of data into a single hierarchy and guarantees the reliability of analysis at any level. The proposed method of object relations has been applied at several full-scale production sites, thereby confirming its stability and operability. The paper provides an analysis of key performance indicators of the application of the method of object relations to solve the problem of forming a technological passport of a product at the steel production, and also discusses ways of further development.

keywords: object relation technique, industry 4.0, Big Data, digital production solutions.

Natalya Rezova[1], **Lev Kazakovtsev** [1,2], **Ivan Rozhnov**[1,2] **and Guzel Shkaberina**[1,2]

HYBRID ALGORITHMS WITH ALTERNATIVE EMBEDDED LOCAL SEARCH SCHEMES FOR THE P-MEDIAN PROBLEM

[1] Reshetnev Siberian State University of Science and Technology, Russia
[2] Siberian Federal University, Russia
levk@bk.ru, {ris2005, z_guzel}@mail.ru

Progress in the development of location theory and clustering methods, the most popular of which are based on solving p-median and similar problems (k-means, k-medoid), is mainly aimed at improving the performance of algorithms. We study the continuous p-median problem with the Euclidean metric. To solve it, we use various algorithmic combinations of the ALA procedure (ALA - Alternate Location-Allocation, Lloyd's algorithm) embedded into more sophisticated algorithmic combinations including the algorithms with greedy ag-glomerative heuristic procedures. In the first step, the ALA algorithm solves a simple discrete optimization problem. The second step solves a series of

contin-uous optimization problems (Weber problems solved by Weiszfeld iterative pro-cesure) that minimize the total distance from the cluster center to its data points. We use a parallel implementation of the ALA algorithm. We study the issue of choosing from two strategies: solve the Weber problem until a given accuracy ε is achieved at each iteration of the ALA algorithm (first approach), or perform only one iteration of the Weiszfeld algorithm at each iteration of the ALA (second approach). Computational experiments on problems of large and medium scale have shown that when choosing a strategy, it is necessary to take into account the complexity of the main algorithm in which the ALA procedure is embedded.

keywords: p-median problem, local search algorithms, Weber problem.

Roman Rudakov[1,2], **Yuri Ogorodnikov**[1,2], **Daniil Khachai**[3] **and Michael Khachay**[1]

A branch-and-price approach for the reliable production process design problem

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
[2] Ural Federal University, Russia
[3] KEDGE Business School, France
r.a.rudakov@gmail.com, {yogorodnikov, nmkhachay}@imm.uran.ru, daniil.khachai@kedgebs.com

Nowadays, designers of manufacturing processes are faced with numerous challenges including insufficient resilience of global supply chains, unpredictable delivery delays, lacks in energy and other vital resources, etc. Many research papers develop models and techniques to fight these challenges, mainly in the field of stochastic models and processes. Nevertheless, the design of production processes that are independent on such uncertainties and mitigate the related risks still remains weekly studied. In this paper, we introduce a novel mathematical model specifying the design of a production process reliable to possible failures of manufacturing units, transportation hubs and links between them, and ensuring a guarantied low level of the related expenses. We call this model the Reliable Production Process Design Problem (RPPDP). It is easy to show, the RPPDP is strongly NP-hard even in the case of the single type of production units and unit capacity of all transportation hubs. By extending the results obtained recently for the Constrained Shortest Path Tour Problem (CSPTP), we propose Mixed Integer Programming (MIP) formulations of our problem, the first Branch-and-Price to find its optimal and sub-optimal solutions. The performance of the proposed algorithm is proved by the results of numerical evaluation.

keywords: branch-and-price, Reliable Production Process Design Problem, Constrained Shortest Path Tour Problem, MIP.

Elena Sofronova[1]

MULTI-OBJECTIVE OPTIMIZATION PROBLEM FOR TRAFFIC CONTROL

[1] Federal Research Center for Computer Science and Control, Russia

Traffic flow in urban road network is a complex control object influenced by various internal and external factors. The mathematical model of control object is given as a system of recurrent finitedifference equations. The multi-objective optimization problem for traffic flows is considered. The control is a vector that determines the moments of switching the working phases of traffic lights at intersections. In multiobjective optimization the solution belongs to the Pareto set. The article for Pareto approximation evolutionary methods are applied. An example of solving the multi-objective optimization problem for real intersections in Moscow is given.

keywords: traffic flow control, optimal control, multi-objective optimization.

Arseniy A. Spiridonov[1] **and Sergey Kumkov**[1]

MULTI-TRAJECTORY CASCADE MERGING OF AIRCRAFT FLOWS

[1] Krasovsky Institute of Mathematics and Mechanics, Russia

sskumk@gmail.com

The problem of creating non-conflict schedule of aircraft arrivals to a given final checkpoint for several incoming aircraft flows in the case of multi-trajectory and multi-stage merging is considered. Here, "multi-stage" means the sequential passage by an aircraft of several points of merging with other aircraft flows. "Multi-trajectory" means that an aircraft may have several routes of movement, leading it to the final point. The assumptions taken in the problem make it possible to consider real schemes of air routes both for area dispatching and for dispatching in airport zones. A formalization of the problem is suggested in the framework of the mixed integer linear programming approach. Certain samples are solved by means of the optimization library Gurobi. The results of numerical simulation and the performance of the computational procedure for model examples and for Koltsovo airport are presented.

keywords: aircraft flows merging, minimal sate interval, non-conflict queue, variation interval, mixed integer linear programming, aircraft order.

Daniil Yudakov[1,2]**, Dmitrii Kolosov**[1] **and Evgeny Bobrov**[2]

MAPPER SIDE GEOMETRIC SHAPING FOR QAM CONSTELLATIONS IN 5G MIMO WIRELESS CHANNEL WITH REALISTIC LDPC CODES

[1] Lomonosov Moscow State University, Russia

[2] Huawei Technologies, Russian Research Institute, Russia

{d.yudakov43, kolosov}@gmail.com, eugenbobrov@ya.ru

In wireless communication systems, there are many stages for signal transmission. Among them, mapping and demapping convert a sequence of bits into a

sequence of complex numbers and vice versa. This operation is performed by a system of constellations - by a set of labeled points on the complex plane. Usually, the geometry of the constellation is fixed, and constellation points are uniformly spaced, e.g., the same quadrature amplitude modulation (QAM) is used in a wide range of signal-to-noise ratio (SNR). By eliminating the uniformity of constellations, it is possible to achieve greater values of capacity. Due to the current standard restrictions, it is difficult to change the constellation both on the mapper or demapper side. In this case, one can optimize the constellation only on the mapper or the demapper side using original methodology. By the numerical calculating of capacity, we show that the optimal geometric constellation depends on SNR. Optimization is carried out by maximizing mutual information (MI). The MI function describes the amount of information being transmitted through the channel with the optimal encoding. To prove the effectiveness of this approach we provide numerical experiments in the modern physical level Sionna simulator using the realistic LDPC codes and the MIMO 5G OFDM channels.

keywords: wireless, MCS, constellation, geometric shaping, BLER, SE, BICM, mutual information, QAM.

Alexander Zyryanov[1], **Sergey Lavlinskii**[1] **and Yury Kochetov**[1]

A BILEVEL PLANNING MODEL FOR PUBLIC-PRIVATE PARTNERSHIP WITH BUDGET UNCERTAINTY

[1] Sobolev Institute of Mathematics, Russia

alexander.zyryanov44@gmail.com, {lavlin, jkochet}math.nsc.ru

We introduce a bilevel model for the strategic planning of the public-private partnership for the case where the budgets at the upper level (government) are uncertain. We know the desire total profit at the upper level and wish to find the most stable solution according the threshold robustness approach. We present the $0-1$ linear bilevel mathematical model for this case and show that this new problem is Σ_2^p-hard in cooperative and non-cooperative cases. Moreover, we claim that the absolute and relative gaps between the optimal cooperative and non-cooperative solutions can be arbitrarily large. We design a Tabu Search algorithm for the problem where the optimal solution of the lower level problem is obtained by the commercial solver. Computational results for real-world instances for the Transbaikalian polymetallic fields are discussed.

keywords: bi-level programming, metaheuristic, computational complexity, Stackelberg game.

8. Game Theory

Julia Chirkova[1]

EQUILIBRIUM ARRIVALS TO PREEMPTIVE QUEUEING SYSTEM WITH FIXED REWARD FOR COMPLETING REQUEST

[1] Institute of Applied Mathematical Research of Karelian Research Centre, Russia
julia@krc.karelia.ru

This paper considers a single-server queueing system in which players send requests into the system with preemptive access and fixed reward for completing request. When a request enters the system, the server immediately starts its service. The current request leaves the system when its service is completed or when the next request comes into the system. We study the non-cooperative game, where each player decides when to arrive at the queueing system within a certain period of time. The objective of the player is to serve his request completely or to maximize the time of receiving service. We investigate Nash equilibrium properties in this game. Finally, we present results of numerical experiments demonstrating the equilibria with different values of the model parameters.

keywords: queueing system, preemptive access, strategic users, optimal arrivals, Nash equilibrium, fixed reward.

This research was supported by the Russian Science Foundation, grant no. 22-11-20015, https://rscf.ru/project/22-11-20015/, jointly with support of the authorities of the Republic of Karelia with funding from the Venture Investment Foundation of the Republic of Karelia.

Mikhail I. Gomoyunov[1,2]

ON OPTIMAL POSITIONAL STRATEGIES IN FRACTIONAL OPTIMAL CONTROL PROBLEMS

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
[2] Ural Federal University, Russia
m.i.gomoyunov@gmail.com

We consider an optimal control problem for a dynamical system described by a Caputo fractional differential equation and a Mayer cost functional. We propose a new method for constructing an optimal positional control strategy, which allows us to generate near optimal controls for any initial system state by using time-discrete recursive feedback control procedures. The method is based on the knowledge of the optimal result functional and uses a special Lyapunov–Krasovskii functional.

keywords: fractional differential equation, optimal control problem, positional control strategy, Lyapunov–Krasovskii functional..

This work was supported by the Russian Science Foundation, project no 21-71-10070, https://rscf.ru/en/project/21-71-10070/

Igor' V. Izmest'ev[1,2] **and Viktor I. Ukhobotov**[1,2]

On a single-type differential game of retention in a ring

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
[2] Chelyabinsk State University, Russia
j748e8@gmail.com, ukh@csu.ru

In a normed space of finite dimension, we consider a single-type differential game. The vectograms of the players are described by the same ball with different time-dependent radii. The motion is constructed using polygonal lines. The target set is determined by the condition that the norm of the phase vector belongs to a segment with positive ends. In this paper, a set defined by this condition is called a ring. The goal of the first player is to retain the phase vector in the ring until a given time. The goal of the second player is the opposite. We have found the necessary and sufficient conditions of retention and constructed the corresponding controls of the players.

keywords: control, differential game.

The research was supported by the Russian Science Foundation, grant no 19-11-00105, https://rscf.ru/project/19-11-00105/.

Vladimir Mazalov[1,2] **and Anna Ivashko**[1,3]

Harmonic numbers in Gambler's ruin problem

[1] Institute of Applied Mathematical Research, Karelian Research Center, Russia
[2] Saint Petersburg State University, Russia
[3] Petrozavodsk State University, Russia
{vmazalov, aivashko}@krc.karelia.ru

The gambler's ruin problem is studied. At each of n steps, the probability that the player wins at the next step depends on the win/lose ratio in previous steps. The player's payoff and the asymptotic formula for large game durations were determined. The numerical results of payoff simulation for different n values are reported.

keywords: random walk, Gambler's ruin problem, ruin probability, reflection principle.

The research was supported by the Russian Science Foundation, grant no 22-11-00051, https://rscf.ru/project/22-11-00051.

Vladimir Mazalov[1,2] **and Anna Rettieva**[1,2]

Exploitation and recovery periods in dynamic resource management problem

[1] Institute of Applied Mathematical Research, Karelian Research Center, Russia
[2] Saint Petersburg State University, Russia
{vmazalov, annaret}@krc.karelia.ru

Dynamic game related to resource management problem is considered. The planning horizon is assumed to be divided into the periods of exploitation

where many players use a common resource and the periods of recovery where the resource stock is evolving according to the natural growth rule. Both noncooperative and coordinated players' behaviors are investigated. The conditions linking the values of exploitation and recovery periods in order to maintain the sustained resource usage are determined. To illustrate the presented approaches, a dynamic bioresource management problem (harvesting problem) with many players and compound planning horizon is investigated.

keywords: dynamic games, resource management problem, exploitation period, recovery period.

Leon Petrosyan[1], **David Yeung**[2] **and Yaroslavna Pankratova**[1]

POWER DEGREES IN DYNAMIC MULTI-AGENT SYSTEMS

[1] Saint Petersburg State University, Russia
[2] Hong Kong Shue Yan University, China
l.petrosyan@spbu.ru

Dynamic multi-agent systems connected in network are considered. To define the power of each agent the analogue of characteristic function is introduced. The values of this characteristic function for each coalition (subset of agents) are calculated as joint payoff of players from this coalition plus payoffs (multiplied on some discount factor) of players which do not belong to the coalition S but have connections with players from S. We suppose that the dynamic of the system is prescribed (this maybe cooperation, Nash equilibrium or any other behaviour). Thus, the characteristic function is evaluated along the prescribed trajectory of agents. And it measures the worth of coalitions under the motion along this trajectory instead of under minimax confrontation or the Nash non-cooperative stance. As solution we consider the proportional solution and introduce Power degrees of an agent based on proportional solution. It is shown that the Power degree (PD) belongs to the Core. PD rank agents according to their importance.

keywords: multi-agent system, proportional solution, power degree.

The research was supported by the Russian Science Foundation, grant no 22-11-00051, https://rscf.ru/en/project/22-11-00051/.

Shimai Su[1] **and Elena M. Parilina** [1]

TRADE-OFF MECHANISM TO SUSTAIN COOPERATION IN POLLUTION REDUCTION

[1] Saint Petersburg State University, Russia
st073379@student.spbu.ru, e.parilina@spbu.ru

We consider an asymmetric differential game of pollution control with two players: developed and developing countries. The vul- nerable tolerance to

the pollution problem distinguishes a developed country from a developing one. This internal characteristic pushes the former to persuade the latter to decrease the polluting production or activities through a contract since the developed country deals with pol- lution reduction alone in the noncooperative setting. However, concern- ing the necessary and actual participation of the developing country in a pollution disposal, in this paper, we introduce a trade-off mechanism by trading partial workload of a pollution disposal between the devel- oped country and the developing one. In return, the developed country shares its profit with the developing country to make the trade work. Finally, the numerical example demonstrates the explicit performance of the proposed trade-off mechanism. We also investigate the efficiency of the trade-off mechanism compared with the fully cooperative and non- cooperative cases.

keywords: differential game, pollution control, trade-off mechanism, partial workload, Pareto optimality..

Anna Tur[1] **and Leon Petrosyan**[1]

COMMUNICATION RESTRICTION-BASED CHARACTERISTIC FUNCTION IN DIFFERENTIAL GAMES ON NETWORKS

[1] Saint Petersburg State University, Russia

{a.tur, l.petrosyan}@spbu.ru

A class of differential games is considered. It is assumed that the players connected by a network interact along paths on this network. It is also supposed that players have the ability to restrict other players' communication. To construct cooperative solutions, a special type of characteristic function is proposed that takes into account the network structure of the game and various types of communication restrictions. The properties of the characteristic function are investigated. The formula for the Shapley value is obtained.

keywords: differential game, cooperative game, network game, characteristic function, Shapley value.

The work of the second author was supported by the RSF, grant no. 22-11-00051, https://rscf.ru/en/project/22-11-00051/.

Rentsen Enkhbat[1], **Pavel Sorokovikov**[2], **Tatiana Zarodnyuk**[2] **and Alexander Gornov**[2]

THE MODIFIED PARALLEL TANGENT GLOBAL SEARCH ALGORITHM FOR FINDING ANTI-BERGE EQUILIBRIUM IN BIMATRIX GAME

[1] Institute of Mathematical and Digital Technology, Mongolia

[2] Matrosov Institute for System Dynamics and Control Theory, Russia

gornov.a.yu@gmail.com

We examine the problem of finding an Anti-Berge equilibrium in bimatrix game

based on a global search algorithm. Finding Anti-Berge equilibrium equivalently reduces to a quadratic programming with an indefinite matrix and linear constraints which belongs to a class of global optimization. To solve the problem numerically, we develop a modified parallel tangent algorithm. The proposed algorithm uses the one-dimensional nonlocal search procedure based on the Strongin and parabolas methods. Stopping criteria of the algorithm is the sufficient condition of Anti-Berge equilibrium. The proposed algorithm is implemented and numerically tested on a collection of bimatrix games.

keywords: anti-Berge equilibrium, bimatrix game, global search algorithm, quadratic programming, global optimization.

Vasily Gusev[1]

TRANSVERSAL PROBLEMS AND STABILITY OF COALITION STRUCTURES

[1] HSE University, Russia

Many actors in the economy, management, sociology, and other spheres have qualifications. Agents develop in a certain direction to become specialists. To be able to implement some projects or assignments, however, representatives of different qualifications or groups have to form coalitions by collaborating. Each coalition member performs the job he/she specializes in. Then, this system of distinct representatives gets a certain utility. The system of distinct representatives, or the transversal, is a coalition of agents from different groups. The transversal value of a cooperative game with coalition structure is introduced to solve the coalition formation problem. It is demonstrated that any cooperative game has a partition that is simultaneously Nash stable, permutation stable, and has a total payoff maximization for the transversal value. The transversal value expresses the players' payoff functions in the workgroup formation game and the game of chairpersons. A player's payoff in the workgroup formation game is a payment for the projects the player participated in. In the game of chairpersons, each player is interested in being their group's leader. It is demonstrated that in such games there exist punctually stable coalition structures.

keywords: coalition formation, Nash stability, transversal.

Yulia Kareeva[1], **Artem Sedakov**[1] **and Mengke Zhen**[1]

STACKELBERG SOLUTIONS IN AN OPINION DYNAMICS GAME WITH STUBBORN AGENTS

[1] Saint Petersburg State University, Russia
a.sedakov@spbu.ru

The paper examines an opinion dynamics game with two influence nodes based on the Friedkin-Johnsen model. In the game, we assume sequential announce-

ments of efforts by the influence nodes and characterize the Stackelberg solutions. We then analyze the solutions by introducing a number of measures that quantify them in different aspects: (i) the role of the information structure, i.e., open-loop vs. feedback, (ii) the advantage of sequential over simultaneous moves, and (iii) whether being a leader in the game is more cost-effective than being a follower. Finally, we perform numerical simulations for Zachary's karate club network to understand how the Stackelberg solutions are sensitive to a change in a parameter characterizing the stubbornness of agents to their initial opinions.

keywords: social networks, opinion dynamics, leadership, Friedkin-Johnsen model, discrete-time games, Stackelberg solution.

This research was supported by the RSF, grant no. 22-21-00346, https://rscf.ru/en/project/22-21-00346/.

Konstantin Kudryavtsev[1]

GUARANTEED EQUILIBRIA IN GAMES UNDER FUZZY INFORMATION ABOUT AN UNCONTROLLED FACTOR

[1] South Ural State University, Russia

In the paper, we consider a non-cooperative N-person game. In this game, the payoffs of the players depend on both a strategy profile and an uncontrolled factor. Players know only fuzzy information about a possible implementation of this uncontrolled factor. For this game, a Pareto guaranteed Nash equilibrium in the strong (and weak) sense is proposed in the paper. As example, a two-person linear-quadratic game under fuzzy information was considered.

keywords: fuzzy game, guaranteed equilibrium, Nash equilibrium.

Denis Kuzyutin[1,2], **Nadezhda Smirnova**[2] **and Yaroslavna Pankratova**[1]

SUSTAINABLE COOPERATION IN MULTI-OBJECTIVE DYNAMIC GAME OF RENEWABLE RESOURCE EXTRACTION

[1] Saint-Petersburg State University, Russia

[2] National Research University Higher School of Economics, Russia

d.kuzyutin@spbu.ru,y.pankratova@spbu.ru,nvsmirnova@hse.ru

Dynamic competitive models of renewable resource extraction management (the "fish war" games are the most famous) have been an active research area (both in non-cooperative and cooperative settings) for few decades. The general assumption in those models is that each player (firm, country, e.t.c.) aims to maximize some economic objective, therefore the unicriterion (scalar) game framework has been employed, while the environmental concern is incorporated in model only through the dynamics of the resource stock evolution or the terminal payoff which depends on the residual stock (scrap value). As it is known,

the multi-objective approach could better describe the players' behavior in real-life applications. However, there are few attempts to apply multicriteria games to dynamic renewable resource extraction models. In the report, we extend the great fish war model by adding additional especially environmental objective to all the players' specification. We accept the linear resource stock dynamics, focus on the two player finite-horizon game with feedback information structure and regard possible players' asymmetry by using different discount factors, parameters to estimate scrape value and weighting coefficients, which the player assigns to the objectives. The aim of the research is to derive non-cooperative and cooperative solution for a bicriteria model, study the properties of these solutions and design a mechanism for sustainable long-term cooperation.

The contributions of the paper is twofold:
(i) we derive analytical solutions, namely a subgame perfect Pareto equilibrium and cooperative solution for bicriteria finite-horizon multistage game of renewable resource extraction with environmentally concerned players;
(ii) we provide a mechanism for sustainable long-term cooperation when the payoff transfers between the players are permitted only within the economical criterion.

keywords: dynamic game, multi-objective game, Pareto equilibrium, cooperative solution, renewable resource extraction, payoff distribution procedure.
The reported study was funded by RFBR and DFG, project number 21-51-12007.

Andrei Neverov[1] **and Olga Krivorotko**[1]

NUMERICAL SOLUTION OF MEAN-FIELD GAME PROBLEM IN EPIDEMIOLOGY OF COVID-19

[1] Sobolev Institute of Mathematics, Russia
a.neverov@g.nsu.ru,o.i.krivorotko@math.ncs.ru

The problem of reconstructing a Hamiltonian of mean-field game (MFG) problem with given distribution of large amount of individuals in differential game in finite time period is investigated for epidemiology. Problem of optimal dynamics of individuals distribution is reduced to solution of system of two PDEs in direct and inverse time. Assuming cost functional to be convex and given form, we try to find its unknown coefficients. We propose numerical algorithm based on collocation method for problem solving in direct and inverse time simultaneously. Inverse problem is reduced to minimisation of fidelity functional, what measures distance between given measurements of individuals distribution and computed one as a result of solution of direct problem with approximated parameters. Algorithm for minimisation of fidelity functional was developed based on gradient descent approach. Gradient is calculated in terms of Frechet derivative based on solution of adjoint problem. The numerical experiments for simple SEIR-HCD model that describes evolution of COVID-19 propagation are demonstrated and discussed.

keywords: mean field games, Kolmogorov-Fokker-Planck equations, Hamilton-Jacobi-Bellman equation, epidemiology, collocation method, SEIR-HCD model.

Andrei Orlov[1]

LOCAL SEARCH ALGORITHM IN BILEVEL OPTIMIZATION PROBLEMS WITH A BIMATRIX GAME AT THE LOWER LEVEL AND ITS TESTING

[1] Matrosov Institute for System Dynamics and Control Theory, Russia
anor@icc.ru

This paper addresses one class of bilevel optimization problems (BOPs) in optimistic statement with an equilibrium at the lower level. Namely, we study BOPs with a convex quadratic objective function under linear constraints at the upper level and a parametric bimatrix game at the lower one, where we need to find a Nash equilibrium point. It is well known that the problem of finding a Nash equilibrium in a bimatrix game is equivalent to the special nonconvex bilinear optimization problem. Nevertheless, we can use an approach developed earlier to transform the original bilevel problem into a single-level nonconvex optimization problem by replacing the lower level with its optimality conditions. Then we are able to apply the Exact Penalization Theory and Global Search Theory (GST) for general d.c. optimization problems to the resulting problem. The principal element of the GST is a special method of local search, which takes into account the structure of the problem under consideration. We present

one of such methods in detail and check its workability and efficiency during computational simulation.

keywords: bilevel optimization, bilevel problems with an equilibrium at the lower level, optimistic solution, bimatrix game, Nash equilibrium, global search theory, exact penalization theory, local search, computational simulation.

Andrei Orlov[1]

A COMPUTATIONAL STUDY OF HYBRID GENETIC GLOBAL SEARCH ALGORITHM FOR HEXAMATRIX GAMES

[1] Matrosov Institute for System Dynamics and Control Theory, Russia
anor@icc.ru

This work addresses the recent hybrid approach to seeking Nash equilibria in polymatrix games of three players (hexamatrix games). This approach, on the one hand, is based on the reduction of the game to a nonconvex optimization problem with a bilinear structure and applying the Global Search Theory in d.c. optimization proposed by A.S. Strekalovsky. On the other hand, to increase the efficiency of one of the key stages of the Global Search - constructing an approximation of the level surface of a convex function that induces the basic nonconvexity in the problem under study - elements of genetic algorithms are used. We present the Hybrid Genetic Global Search Algorithm (HGGSA) in detail and implement the comparative testing of Basic Global Search Algorithm and HGGSA on the wide field of randomly generated hexamatrix games.

keywords: polymatrix games of three players, hexamatrix games, global search theory, local search, level surface approximation, basic global search algorithm, hybrid genetic global search algorithm, computational simulation, comparative testing.

Stepan Utyupin[1], **Andrey Melnikov**[2] **and Vladimir Beresnev**[2]

REPRESENTATION OF THE ETERNAL VERTEX COVER PROBLEM AS A DYNAMIC STACKELBERG GAME

[1] Novosibirsk State University, Russia
[2] Sobolev Institute of Mathematics, Russia
melnikov@math.nsc.ru, beresnev@math.nsc.ru

The eternal vertex cover problem is a variant of the graph vertex cover problem that can be represented as a dynamic game between two players (attacker and defender) with an infinite number of steps. At each step, there is an arrangement of guards over the vertices of the graph, forming a vertex cover. The attacker attacks one of the graph's edges, and the defender must move the guard along the attacked edge from one vertex to another. In addition, the defender can move any number of other guards from their current vertices

to some adjacent ones to obtain a new vertex cover. The attacker wins if the defender cannot build a new vertex cover after an attack. The eternal vertex cover problem is to determine the smallest number of guards for which the attacker has no sequence of attacks leading to a win. We consider the eternal vertex cover problem as a dynamic Stackelberg game. At each step of the game, a vertex cover is considered and the optimal solution of the bilevel programming problem is calculated, which allows to determine whether the given vertex cover is resistant to attack. The results of the computational experiment on the stability of various vertex covers of random graphs against attacks are presented.

keywords: eternal vertex cover, graph algorithms, graph protection, bilevel programming, dynamic Stackelberg game.

Zeyang Wang[1] **and Hongwei Gao**[1]

MIN-DISTANCE BARGAINING SOLUTION IN DIFFERENTIAL GAMES

[1] Saint Petersburg State University, Russia

In this article, we proposed a new bargaining solution, and call it as "Min-distance bargaining solution". We applied this new solution in differential games model. We presented the algorithm and considered time consistency in the differential games, obtained some interesting results about the new solution in some specific and reality situations. Simulation results were all showed on the resource extraction differential games model. We test the time consistency of the new solution, and we compared our bargaining solution with the existing bargaining solutions such as Nash bargaining solution, Kalai-Smorodinsky solution, Egalitarian solution.

keywords: differential games, bargaining solutions, time consistency.

Jiangjing Zhou[1]**, Ovanes Petrosian**[1] **and Hongwei Gao** [1]

CONTINUOUS BAYESIAN UPDATING IN DIFFERENTIAL GAME MODELS WITH UNCERTAINTIES

[1] Saint Petersburg State University, Russia

We consider a class of differential games with continuous Bayesian updating, in which the dynamic equations of the system contain unknown parameters. In order to obtain the optimal strategy of the player in this type of differential game, first use Bayesian filter to estimate the unknown parameters. The evolution process of expectations obtained from different beliefs is represented by a random process, which has process errors. At the same time, at each moment, the player will receive a measurement of an unknown parameter, but the

measurement has an error, which we use white noise to describe. We can show that, according to the process we construct, the estimated value will approach the true value of the unknown parameter, and we give a Nash equilibrium with continuous Bayesian updating.

keywords: Bayesian filter, continuous Bayesian updating, Nash equilibrium with continuous updating.

Iakov Zhukov[1]

THE PROBLEM OF THE LEADER OF A COALITION PARTITION

[1] Novosibirsk State University, Russia

y.zhukov@g.nsu.ru

We study the problem of the leader of a coalition partition. We proof the existence of a Nash-stable coalitional partition in the problem of the leader of a coalition partition, where, according to the weight rule, firstly the leading coalition is chosen, then the leader of the whole game. The model shows the existence of optimal weights that allow first maximizing the probability of leadership of a coalition of players, and then the probability of leadership of a player in a given partition. We proof the existence of an equilibrium in the modification of the game, where the players have two weights. For a model where the leader of each coalition is selected first in the partition, and then the leader among them, we show the existence of a stable partition for the case when the weights of the players are equal. We show the existence of a stable coalition partition in games with a rank function with a number of specified properties.

keywords: cooperative game theory, group formation, stable partition, pure Nash equilibria, potential games.

9. Optimal Control and Mathematical Economics

Boris Ananyev[1]

GUARANTEED EXPECTATION OF THE GLOCK POSITION WITH RANDOM DISTRIBUTION OF ITEMS

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
abi@imm.uran.ru

We consider estimation problems for a flock of linear systems with random matrices and additive uncertain disturbances. Because of uncertainty we may only assume that the state of each flock's item belongs to some random information set that can be built according to measurement equations. Our goal is to define and investigate a mean value of random information sets. All the sets are described by its support functions. Some theorems on the approximation of the sets by be simpler objects are given. Particular cases, where matrices of the systems or probability spaces have special formes, are elaborated. In conclusion we review some examples.

keywords: guaranteed estimation, information set, set of attainability, averaged controllability and observability.

Yulia Danik[1] **and Mikhail Dmitriev**[1]

THE ALGORITHM FOR THE CONSTRUCTION OF A SYMBOLIC FAMILY OF REGULATORS FOR NONLINEAR DISCRETE CONTROL SYSTEMS WITH TWO SMALL PARAMETERS

[1] Federal Research Center for Computer Science and Control, Russia
yuliadanik@gmail.com, mdmitriev@mail.ru

In this paper, the stabilization problem for a discrete weakly nonlinear system with two small positive parameters and a quadratic quality criterion is considered. The parameters are at the nonlinearities in system matrices and can be of different order. We use the asymptotic methods to find a parametric family of solutions in the form of the state feedback. For the construction of a parametric family of feedback control we apply the Discrete State-dependent Riccati equation (D-SDRE) approach, which consists in solving the corresponding discrete matrix algebraic Riccati equation with state-dependent coefficients. An asymptotic expansion of the solution of the corresponding Riccati equation is found in the form of a power series by two parameters. This regular asymptotic series is used for the construction of a one-point matrix Pade approximation by two parameters. The numerical experiments on a grid of parameters demonstrate the stabilization of the closed-loop systems with the proposed regulators. The resulting Pade controllers have interpolation and extrapolation properties, and often significantly improve the approximation accuracy in comparison with controllers based on the regular asymptotic series by one or two parameters.

keywords: small parameter, pade approimation, D-SDRE approach, asymptotic approximation.

Mikhail Gusev[1] **and Ivan Osipov**[1]

Approximate solution of small-time control synthesis problem based on linearization

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
gmi@imm.uran.ru, i.o.osipov@imm.uran.ru

We consider the problem of a feedback control design for a nonlinear control-affine system. The aim of the control is to bring trajectories of the closed system to the origin of coordinates in a given time, providing the minimal value of an integral functional. The object under study is the nonlinear system, closed by a linear feedback controller. The controller is obtained as a solution of the LQR problem for the linearized system. We indicate sufficient conditions for this linear feedback to give a local solution to the control synthesis problem under consideration. In addition, we give some error estimates for the values of the functional.

keywords: control synthesis, linear feedback, linearization.

The work was performed as part of research conducted in the Ural Mathematical Center with the financial support of the Ministry of Science and Higher Education of the Russian Federation (Agreement no 075-02-2023-913)

Alexander Fominyh[1]

Method for solving a differential inclusion with a subdifferentiable support function of the right-hand side

[1] Saint Petersburg State University, Russia
alexfomster@mail.ru

The paper studies differential inclusions such that the sup- port function of the set on the right-hand side considered at each time moment is subdifferentiable. It is required to find a trajectory satisfying the differential inclusion as well as the boundary conditions and some phase constraints. The applied problems are given where such systems arise. The problem is reduced to minimizing the functional. The superdif- ferentiability of this functional is proved, minimum conditions (in terms of superdifferential) are obtained. The superdifferential descent method is described; the convergence of the method to a stationary point of the functional is proved. An illustrative example is given.

keywords: differential inclusion, support function, superdifferential.

Danis Ibragimov[1] **and Sofya Guseva**[1]

A priori estimates of the objective function in the speed-in-action problem for a linear two-dimensional discrete-time system

[1] Moscow Aviation Institute, Russia
rikk.dan@gmail.com

The two-dimensional linear system with discrete time and bounded control is considered. It is assumed that the system matrix is nondegenerate and diagonalizable by means of a similarity transformation and the set of admissible values of the control is convex and compact. The speed-in-action problem is studied for a given system, in particular, it is required to construct a priori estimates of the optimal value of the objective function in the speed-in-action problem as a function of the initial state and parameters of the system, which do not require an exact construction of the class of 0-controllable sets. In the trivial case, when the system matrix equals its real Jordan form, and the set of admissible values of the control is either a ball or a rectangle, the optimal value of the objective function in the speed-in-action problem is obtained explicitly. The method is proposed, which reduces case of an arbitrary control system to a trivial case by means of the singular decomposition of the system matrix and constructing internal and external approximations of constraints on control actions. Numerical calculations are presented that demonstrate the efficiency and accuracy of the developed technique.

keywords: linear discrete-time system, speed-in-action problem, optimal control, preliminary estimates of the optimum.

Supported by RSF, grant no 23-21-00293.

Pavel Lebedev[1] **and Alexander Uspenskii**[1]

Analytical construction of the singular set in one class of time-optimal control problems in the presence of linear segments of the boundary of the target

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
pleb@yandex.ru

A time-optimal control problem with a spherical velocity vectogram is considered. For one class of non-convex planar target sets with a part of their boundary coinciding with a line segment, conditions are found to construct branches of singular (scattering) curves in analytical form. Explicit formulas are obtained for pseudo-vertices, i.e., singular boundary points of the target set generating branches of the singular set. An analytical relation is revealed between the endpoints of different optimal trajectories with the same initial conditions on the singular set that falls on the target set in a neighborhood of a pseudo-vertex. Formulas are found for the extreme points of the singular set branches. The developed approaches to constructing exact non-smooth solu-

tions for dynamic control problems are illustrated with examples.

keywords: scattering curve, singular set, pseudo-vertex, mapping, curvature.

Valeriy Marakulin[1]

ON THE EXISTENCE OF FUZZY CONTRACTUAL ALLOCATIONS, FUZZY CORE AND PERFECT COMPETITION IN AN EXCHANGE ECONOMY

[1] Sobolev Institute of Mathematics, Russia

marakulv@gmail.com

The fuzzy core is well-known in theoretical economics, it is widely applied to model the conditions of perfect competition. In contrast, the original author's concept of fuzzy contractual allocation as a specific element of the fuzzy core is not so widely known in the literature, but it also represents a (refined) model of perfect competition. This motivates the study of its validity: the existence of fuzzily contractual allocations in an economic model; it also implies the existence (non-emptiness) of the fuzzy core and develops a known approach. The proof is based on two well-known theorems: Michael's theorem on the existence of a continuous selector for a point-to-set mapping and Brouwer's fixed point theorem. In literature, only the non-emptiness of the fuzzy core was proven under essentially stronger assumptions — typically, it applies replicated economies and Edgeworth equilibria.

keywords: fuzzy core, fuzzy contractual allocation, Edgeworth equilibria, perfect competition, existence theorems.

The study was supported by the Program of Basic Scientific Research of the Siberian Branch of the Russian Academy of Sciences (grant no FWNF-2022-0019).

Valeriy Rozenberg[1,2]

AN APPROACH TO SOLVING INPUT RECONSTRUCTION PROBLEMS IN STOCHASTIC DIFFERENTIAL EQUATIONS: DYNAMIC ALGORITHMS AND TUNING THEIR PARAMETERS

[1] Krasovsky Institute of Mathematics and Mechanics, Russia

[2] Ural Federal University, Russia

rozen@imm.uran.ru

Within the framework of the key approach from the theory of dynamic inversion, input reconstruction problems for stochastic differential equations are investigated. Different types of input information are used for the simultaneous reconstruction of disturbances in both the deterministic and stochastic terms of the equations. Feasible solving algorithms are designed; estimates of their convergence rates are derived. An empirical procedure adapting an algorithm to a specific system's dynamics to obtain best approximation results is discussed. An illustrative example for this technique is presented.

keywords: stochastic differential equation, dynamic input reconstruction, controlled model, feedback control.

Alexander Shananin[1,2,3,4,5] **and Nikolai Trusov**[1,2,3]

MATHEMATICAL MODELING OF THE HOUSEHOLD BEHAVIOR ON THE LABOR MARKET

[1] Federal Research Center for Computer Science and Control, Russia
[2] Moscow Center of Fundamental and Applied Mathematics, Russia
[3] All-Russian Research Institute of Labor, Russia
[4] Moscow Institute of Physics and Technology, Russia
[5] Peoples' Friendship University of Russia (RUDN), Russia
alexshan@yandex.ru, trunick.10.96@gmail.com

We describe the economic behavior of the household. On the one hand, the household acts as a consumer that seeks to maximize the discounted consumptions on the imperfect lending and saving market. On the other hand, the household acts a worker on the labor market that receives salary and wants to enlarge its competencies to receive higher wages. In this work we present the model of the worker behavior that spends its salary on consumptions and on the investments in human capital. The investments in human capital helps to obtain new skills and increase the qualifications of the employee. This provides an opportunity to receive higher wages. The problem is formalized as an optimal control problem on the infinite time horizon. We introduce its solution in the form of the Pontryagin maximum principle, find the transversality conditions of the conjugate variables, and introduce the identification approach to reproduce the behavior of employees in different social layers based on the Russian Federation Household Budget Survey.

keywords: mathematical modeling, optimal control, infinite time horizon, maximum principle, identification problem.

Supported by RSF (grant No.23-21-00281).

Margarita Sotnikova[1] **and Ruslan Sevostyanov**[1]

VISUAL POSITIONING OF A MOVING OBJECT USING MULTI-OBJECTIVE CONTROL ALGORITHM

[1] Saint Petersburg State University, Russia
m.sotnikova@spbu.ru, sevostyanov.ruslan@gmail.com

The paper is devoted to the problem of multi-objective control design for visual positioning of moving objects. This research area incorporates the methods of control theory and computer vision, and has a special importance for autonomous vehicles, where visual information from on-board camera gives a rich data about the surrounding world. Visual information can be ef-fectively

used in feedback control, for example, in such applications as vis-ual positioning, tracking a visually given line, moving in a changing envi-ronment with visual obstacles. The main objective of this work is to develop the results obtained earlier and design a feedback control algorithm for visual positioning problem based on the multi-objective approach. This approach allows to take into ac-count a set of requirements for closed-loop system performance in different regimes. These regimes, in particular, include the object motion under con-stant or random external disturbances. The multi-objective structure of the control law has adjustable elements that must be selected in accordance with the imposed requirements. It is convenient to formulate the problems of searching these elements as optimization tasks on the corresponding admissible sets. The nonlinear mathematical model of object dynamics is considered. In addition, the nonlinear model of the dynamics in image plane of the camera is introduced. The control objective in visual positioning problem is to provide the desired projection of some three-dimensional object of the scene to the image plane. This projection is described by features vector. The main result of the work is the developed feedback control law, which is based on multi-objective structure and computer vision algorithms. Methods for searching adjustable elements of multi-objective structure are proposed. The efficiency of the approach is illustrated by a computational experiment in MATLAB environment.

keywords: visual positioning, computer vision, camera, multi-objective control, external disturbances, moving object.

Vladimir Ushakov[1], **Aleksandr Ershov**[1,2], **Anna Ershova**[2], **Aleksandr Alekseev**[3]

Linear interpolation of program control with respect to a multidimensional parameter in the convergence problem

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
[2] Ural Federal University, Russia
[3] JSC 'OKB Novator', Russia
ushak@imm.uran.ru, ale10919@yandex.ru
anya.erygina@yandex.ru, sztern987@gmail.com

We consider a control system containing a constant three-dimensional vector parameter, the approximate value of which is reported to the control person only at the moment of the movement start. The set of possible values of unknown parameter is known in advance. An convergence problem is posed for this control system. At the same time, it is assumed that in order to construct permissive control, it is impossible to carry out cumbersome calculations based on the pixel representation of reachable sets in real time. Therefore, to solve the convergence problem, we propose to calculate in advance several resolving

controls, corresponds to possible parameter values in terms of some grid of nodes. If at the moment of the movement start it turns out that the value of the parameter does not coincide with any of the grid nodes, it is possible to calculate the program control using the linear interpolation formulas. However, this procedure can be effective only if a linear combination of controls corresponding to the same "guide" in the terminology of N.N. Krasovskii's Extreme Aiming Method is used. In order to be able to effectively apply linear interpolation, for each grid cell, we propose to calculate 8 'nodal' resolving controls and use the method of dividing control into basic control and correcting control in addition. Due to the application of the latter method, the calculated solvability set turns out to be somewhat smaller than the actual one. But the increasing of accuracy of the system state transferring to the target set takes place.

keywords: control system, convergence problem, unknown parameter, program control, linear interpolation.

This research was supported by the RSF, grant no 19-11-00105, https://rscf.ru/en/project/19-11-00105/).

Anastasiia Usova[1] **and Alexander Tarasyev**[1,2]

BEHAVIOR OF STABILIZED TRAJECTORIES OF A TWO FACTOR ECONOMIC GROWTH MODEL UNDER THE CHANGES OF A PRODUCTION FUNCTION PARAMETER

[1] Krasovsky Institute of mathematics and Mechanics, Russia

[2] Ural Fedelal University, Russia

ausova@imm.uran.ru, tam@imm.uran.ru

Based on a two-factors economic growth model with a production function of a constant elasticity of substitution, the paper considers a control problem with the infinite time interval and analyzes its stabilized solutions, when the elasticity parameter changes. A qualitative analysis of a Hamiltonian system reveals an existence of a saddle steady state, which continuously depends on the elasticity coefficient. In the domain containing the steady state, the stabilization of a Hamiltonian system is performed, and solutions of the stabilized system are numerically constructed. Varying the elasticity coefficients of CES-production function, these solutions undergo changes. The paper shows that for a limit value of the elasticity parameter, when a production function turns into the Cobb-Douglas production function, corresponding stabilized solutions converge to the limit case associated with the Cobb-Douglas function. Numerical experiments support the theoretical conclusions.

keywords: Hamiltonian systems, steady state, sensitivity analysis, stabilizer, production function.

The research of the first author, Anastasiia A. Usova, is supported by the RSF, grant no 19-11-00105), https://rscf.ru/project/19-11-00105/.

Sergey Antsyz[1]

ON THE SEARCH FOR THE HIGHEST LEVEL OF WELLFARE OF AN ECONOMIC ENTITY

Sobolev Institute of Mathematics, Russia
antzys@math.nsc.ru

The object of the study is a dynamic management model of an economic entity. A subject's development strategy is called rational if his wellfare exceeds a given level during each interval of the period under consideration. In [1], based on the Solow equation [2], a new model was proposed for finding rational strategies that, when forecasting for the medium term, unexpectedly surpass the strategies obtained by the classical Ramsey-Kass-Koopmans model [3]. To substantiate this statement, an algorithm of actions of the social planner was developed, described in the presentation [4]. In the present paper, it is proposed to provide a detailed description of the first stage of this algorithm and establish the relationship of the maximum level of wellfare with the observed parameters of the functioning of an economic entity: its initial capital, inflation rate, etc.

[1] Antsyz S.M.: On Refinement of the Simplest Growth Model // 17th International Asian School-Seminar Optimization Problems of Complex Systems (OPCS), Moscow, Novosibirsk (Russia), Almaty (Kazakhstan). IEEE Xplore, 2021. – P. 9-12.

[2] Solow R.M.: A Contribution to the Theory of Economic Growth // The Quarterly Journal of Economics, vol. 70, is. 1, pp. 65-94, February 1956.

[3] Romer D.H.: Advanced macroeconomics, 4th ed. p. cm., University of California. Berkeley: McGraw-Hill, 2012.

[4] Antsyz S.M.:On the two-stage construction of a rational strategy of an economic entity

keywords: growth economic models, highest level of wellfare, social planner's strategy, initial capital, inflation rate.

The author was supported by the Ministry of Science and Higher Education of the Russian Federation, grant no FWNF–2022–0019).

Igor Bykadorov[1]

UNIMODALITY OF EQUILIBRIUM WELFARE IN INTERNATIONAL TRADE UNDER MONOPOLISTIC COMPETITION

[1] Sobolev Institute of Mathematics, Russia
bykad@math.nsc.ru

We study the classical monopolistic competition trade model. The utility of consumers is additively separable with pro-competitive sub-utility function, the production costs are linear, and the transport cots are "of iceberg type". We examine the local comparative statics of market equilibrium with respect to transport costs. Earlier we found the following results: (1) (intuitive!) an increase w.r.t. transport costs near free trade leads to a decrease in social welfare; (2) (counter-intuitive!) an increase w.r.t. transport costs near autarky leads to an increase in social welfare. Thus, social welfare is non-monotonic. In this regard, a natural question arises about the unimodality of social welfare. We show the unimodality of social welfare for an important class of elementary utility function, namely, for the Behrens-Murata function.

keywords: monopolistic competition, international trade, market equilibrium, comparative statics, free trade, autarky, unimodality of welfare.

Viktor Chistyakov[1]

INFLUENCE OF SINGULAR POINTS OF THE JACOBI EQUATION ON THE PROPERTIES OF THE DEGENERATE QUADRATIC FUNCTIONAL

[1] Matrosov Institute for System Dynamics and Control Theory, Russia
chist@icc.ru

In recent decades, a large number of publications have addressed the properties of integral functionals with a violated strengthened Legendre condition at individual points or on sets of positive measure of the segment where the integrand is defined. This paper presents the latest results on quadratic functionals defined on the set of continuously differentiable vector functions that vanish at the ends of the integration interval. It is assumed that:
1. The strengthened Legendre condition is violated at all points of the integration interval;
2. There are singular points (turning points) of the Jacobi vector equation on the integration interval.
The main objective of the work is to the search for conditions under which:
1) space the kernel of the functional is finite-dimensional, in particular, zero;
2) the quadratic functional is non-negative in the domain;
3) small changes in the functional correspond to small changes in the argument in some norms.

keywords: quadratic functional, Legendre condition, Jacobi equation, singular point.

Elena Khoroshilova[1] **and Anatoly Antipin**[2]

A proven method for an optimal control problem with linear dynamics, phase constraints, and boundary value problems

[1] Lomonosov Moscow State University, Russia
[2] Federal Research Center for Computer Science and Control, Russia
khorelena@gmail.com, asantip@yandex.ru

On a finite time interval, we consider an optimal control problem with linear dynamics and a tube of phase constraints. Constraints are a continuous system of linear inequalities depending on time. At each moment of time we have in section a polyhedron in a finite-dimensional space R^n. At the ends of the time interval, in n-dimensional spaces, objective functions are set. These functions, together with constraint polytopes, generate two boundary value problems of linear programming. The dynamics includes two time-dependent functional variables (control and phase variables), as well as finite-dimensional variables of boundary value problems. Dynamics develops on a time interval in Hilbert space. For each control taken from the convex closed set of the Hilbert space, there is a unique phase trajectory. This correspondence is guaranteed by the existence and uniqueness theorem for the solution of a differential equation. The statement of the problem involves choosing a control so that the corresponding phase trajectory is to be inside the phase tube (perhaps, touching its walls), and at the ends of the segment, the phase trajectory takes values that coincide with the solutions of boundary-value problems of linear programming. To solve the problem, an evidential method of the saddle type is proposed. This method (computing technology) generates a sequence of asymptotic approximations to the solution of the problem. This sequence has limit points, which are guaranteed to be solutions to the source problem. The proof of the method convergence to the solution is based on the theory of duality, the Lagrange function, and the convexity of the problem. Only evidence-based calculations transform mathematical models into a tool for obtaining reasonable estimates for making guaranteed solutions.

keywords: optimal control, phase constraints, duality, Lagrangian formalism, boundary value problems, saddle methods, convergence.

Suriya Kumacheva[1], **Ekaterina Zhitkova**[1] **and Galina Tomilina**[2]

The impact of public attitudes to vaccination on influenza epidemic dynamics: societal and economic risks

[1] Saint Petersburg State University, Russia
[2] EPAM Systems, Spain
e.zhitkova@spbu.ru, g.tomilina@yandex.ru

Public opinion about influenza vaccination is influenced by a variety of factors and ranges from strongly negative to strongly positive. The change in the

proportions of the population supporting vaccination and strongly against it, doubting and undecided, can be described as the opinion dynamics in the network, and reaching a stable state – as a consensus in the De Groot's meaning [De Groot M.H., 1974].

The aim of this study is to test the assumption that the dynamics of opinions about vaccine prophylaxis influence the development of the epidemic process. The population is assumed to be divided into risk groups (age, occupational, etc.), and the opinion dynamics are considered for each risk group separately. It is assumed that, as a result of the exchange of information and opinions, each agent makes the final decision on his/her vaccination. Such decisions have an impact on the formation of immune status in each agent personally, as well as on the formation of collective immunity in the population as a whole. The process of opinion dynamics is supposed to be completed prior to the onset of the seasonal influenza outbreak, and each agent determined with a decision to vaccinate by that moment. The development of the epidemic process is described by one of the modifications of the classical Kermack-McKendrick model [Kermack, W.O., McKendrick, A.G., 1927], which takes into account the mortality of agents due to disease. The problem of assessing the economic risks arising from the intensive growth of disease for society as a whole, as well as vaccine prophylaxis as one of the tools for regulating these risks, is investigated. The theoretical study is accompanied by simulation of opinion dynamics and the epidemic process, implemented on a network model based on a random graph. The simulation is carried out using statistical data on the influenza incidence and annual vaccination campaigns in the Russian Federation [The web-site of the Russian Federation State Statistics Service. `http://rosstat.gov.ru/`]. A numerical experiment based on statistical data is conducted and a scenario analysis is performed.

keywords: vaccination, epidemic model, network model, opinion dynamics, economic risks.

Project: Improvement of Insurance Coverage of the Population under Biological Threat (Saint Petersburg State University. Pure ID: 92423693)

Dmitry Makarov[1] **and Mikhail Dmitriev**[1]

ITERATIONS BASED CONTROLLER CONSTRUCTION FOR A WEAKLY NONLINEAR SINGULARLY PERTURBED SYSTEM

[1] Federal Research Center for Computer Science and Control, Russia
`mdmitriev@mail.ru, makarov@isa.ru`

Control synthesis for nonlinear systems is the main goal for many practical applications. One of the approaches to solving such a problem is based on the representation of a nonlinear system in a form formally linear in control and state, but with state-dependent matrices. In this case, the final controller may

be determined by setting a quadratic quality criterion, the weight matrices of which can also depend on the state. This approach is closely related to the SDRE approach and requires the solution of the corresponding matrix Riccati-type differential equation, in which the coefficient matrices are also functions of the state. In this paper, we consider the problem of constructing a controller on a finite time interval for a two-time-scale weakly nonlinear high-dimensional system, the mathematical model of which is represented as a singularly perturbed system, where coefficients are weakly nonlinear state functions. The iterative method is used to solve the Riccati-type equation.

keywords: weakly nonlinear systems, singularly perturbed systems, SDRE approach, iterative methods.

Dmitrii Novikov[1]

ON THE SOLUTION OF THE SIMPLEST TIME-OPTIMAL PROBLEM WITH PHASE CONSTRAINTS

[1] Krasovsky Institute of Mathematics, Russia

ya.novikovdmitry@yandex.ru

In many applied problems of controlling the motion of a rigid body in the atmosphere under the action of gravitational and reactive forces, various problems arise to control its spatial orientation, which is described by dynamic and kinematic Euler equations. In this work, one of such control problems is studied in an extremely simplified formulation. The rotational motion of a rod on a plane around its center of mass is considered under the action of a constant modulo force applied to one of the ends of the rod. As a control parameter, the rate of change of the angle between the rod and the vector that specifies the direction of the specified force is used. Restrictions are imposed on the control and the current phase state of the linear dynamic system describing the motion of the rod. The desired control must satisfy the constraints and ensure the transfer of the system from its initial state to some specified final state in the minimum time with the fulfillment of phase constraints. An approach to constructing the required control based on solving an auxiliary optimal control problem without phase constraints is discussed. The existence and uniqueness of a solution to this problem is proved. The questions of the existence of an optimal control in the time-optimal problem in the presence of phase constraints are discussed. A method for finding a suboptimal control in this problem is proposed. The results obtained are illustrated by examples of the numerical solution of a number of model problems.

keywords: linear dynamic system, optimal control problems, phase constraints, suboptimal control, dynamic and kinematic Euler equations.

Olga Samsonyuk[1]

OPTIMAL CONTROL ALGORITHMS BASED ON FUNCTIONS OF THE LYAPUNOV TYPE

[1] Irkutsk State University, Russia
samsonyuk.olga@gmail.com

This paper addresses nonlocal optimality conditions operating with functions of the Lyapunov type. These optimality conditions are constructive and close to the dynamical programming, where we use monotone solutions to the corresponding Hamilton-Jacobi equation instead of the Bellman function. Based on these results, we propose a computational algorithm for solving an optimal control problem with intermediate state constraints. The algorithm is implemented in the Julia programming language using the built-in JuMP package.

keywords: optimal control, Lyapunov type function, optimality conditions, numerical solution.

Irina Vorobeva[1]

CONVEX OPTIMIZATION PROBLEM FOR REGIONAL ECONOMY MODELING AND COVID-19 INFLUENCE ON IT

[1] Lomonosov Moscow State University, Russia

The article is devoted to the COVID-19 influence on the regional economy modeling of Novosibirsk region. The following model is based on the input-output balance model of the region given the effect of shocks on individual industries. The localization model for modeling regional input and output flows is used. It allows one to construct of input-output tables using regional data on the number of workers in the area and national input-output tables. COVID-19 is modeled as a shock source for the industries. Its influence is evaluated using two approaches. The first one uses regional labour figure dynamics to see how deceaserelated mortality affected the industries involved. The second model is based on localized change in national GDP. The results of shock influence in both cases are compared and analyses.

keywords: input-output tables, localisation coefficients, COVID-19 macroeconomic shocks.

This work is supported by the RSF, grant no. 18-71-10044- P).

Tatiana Zvonareva[1] **and Olga Krivorotko**[1]

NUMERICAL SOLUTION OF THE OPTIMAL CONTROL PROBLEM IN SOCIAL NETWORKS

[1] Sobolev Institute of Mathematics, Russia
t.zvonareva@g.nsu.ru, o.i.krivorotko@math.ncs.ru

The control problem for the density of influenced users in a synthetic online social network is numerically investigated. The optimal solution of the control problem is obtained by the joint solution of the Kolmogorov-Fokker-Planck equation describing the dynamics of the density of influenced users [1], and by the Hamilton-Jacobi-Bellman equation responsible for the optimal strategy. An algorithm based on finite-difference and Lagrangian [2] approaches are developed. The degree ill-posedness of the source problem for fixed control based on the analysis of the degree of decreasing singular numbers of the matrix of the operator of linearized inverse problem. The regularization algorithm of the source problem with optimal control is proposed.

[1] T. A. Zvonareva, O. I. Krivorot'ko, Comparative analysis of gradient methods for source identification in a diffusion-logistic model, Comput. Math. and Math. Phys. 62 (2022), 674-684,
https://doi.org/10.1134/S0965542522040145.

[2] V. Shaydurov, S. Zhang, E. Karepova, Conservative difference schemes for the computation of mean-field equilibria, AIP Conf. Proc. 1895 (2017), 020004,
https://doi.org/10.1063/1.5007358

keywords: Kolmogorov-Fokker-Planck equation, Hamilton-Jacobi-Bellman equation, social networks, source problem, optimal control, mean field games, regularization.

Igor Zykov[1]

ESTIMATION OF REACHABLE SETS OF CONTROL SYSTEMS USING SUPPORT FUNCTIONS

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
zykoviustu@mail.ru

Estimation of reachable sets of control systems using support functions.

In the first part of the report proposes a transition from a continuous linear to a discrete control system by uniform partition the time interval and replacing the controls (geometric and several integral) at the partition step their average values. An algorithm for constructing the boundary of reachable sets based on solving a family of conic programming problems is proposed. In the second part we propose an algorithm for constructing external estimates for reachable sets of nonlinear control systems with geometric constraints on the control and uncertainty in the initial data. The nonlinear function on the right side of the system is assumed to be quadratic in the state variable. A transition from a continuous to a discrete system is proposed using an improved Euler method with further construction of supporting hyperplanes to estimate the reachability set at the current partition step. The result is a series of quadratic programming problems that in general are not convex. To solve the

latter in the convex case the method of conic programming is used and in the non-convex case reduction to problems of mixed-integer linear programming is used. Numerical simulation was carried out.

keywords: control system, approximation, estimation, reachable set, different types constraints, integral constraints, geometric constraints, support functions, mixed-integer linear programming.

10. Optimization in Machine Learning and Artificial Intelligence

Artem Aroslankin[1] **and Valeriy Kalyagin**[1]

UNCERTAINTY OF GRAPH CLUSTERING IN CORRELATION BLOCK MODEL

[1] HSE University, Russia

adaroslankin@gmail.com, vkalyagin@hse.ru

Cluster analysis is a powerful tool in network science and it is well developed in many directions. However, the uncertainty analysis of clustering algorithms is still not sufficiently investigated in the literature. To study uncertainty of clustering algorithms we propose to use a new model, which we call correlation block model. We suggest to measure uncertainty of clustering algorithms by error in cluster identification by observations. Uncertainty of different clustering algorithms are compared using proposed methodology. New and interesting phenomena are observed.

keywords: graph clustering , uncertainty, random variable networks.

This research was supported by Basic Research Program at the National Research University Higher School of Economics (HSE) RSF, grant no 22-11-00073

Kirill Kalmutskiy[1,2]**, Lyailya Cherikbayeva**[3]**, Alexander Litvinenko**[4] **and Vladimir Berikov**[1,2]

MULTI-TARGET WEAKLY SUPERVISED REGRESSION USING MANIFOLD REGULARIZATION AND WASSERSTEIN METRIC

[1] Sobolev Institute of mathematics, Russia

[2] Novosibirsk State University, Russia

[3] Al-Farabi Kazakh National University, Kazakhstan

[4] RWTH Aachen, Germany

berikov@math.nsc.ru, k.kalmutskii@g.nsu.ru

cherikbayeva.lyailya@gmail.com, litvinenko@uq.rwth-aachen.de

In this paper, we consider the weakly supervised multi-target regression problem where the observed data is partially or imprecisely labelled. The model of the multivariate normal distribution over the target vectors represents the uncertainty arising from the labelling process. The proposed solution is based on the combination of a manifold regularisation method, the use of the Wasserstein distance between multivariate distributions, and a cluster ensemble technique. The method uses a low-rank representation of the similarity matrix. An algorithm for constructing a co-association matrix with calculation of the optimal number of clusters in a partition is presented. To increase the stability and quality of the ensemble clustering, we use k-means with different distance metrics. The experimental part presents the results of numerical experiments with the proposed method on artificially generated data and real data sets. The results show the advantages of the proposed method over existing solutions.

keywords: weakly supervised learning, multi-target regression, manifold regularization, low-rank matrix representation, cluster ensemble, co-association matrix.

Tatiana Makarovskikh[1], **Anatoly Panyukov**[1] **and Mostafa Abotaleb**[1]

USING GENERAL LEAST DEVIATIONS METHOD FOR FORECASTING OF CROPS YIELDS

[1] South Ural State University, Russia
Makarovskikh.T.A@susu.ru, paniukovav@susu.ru
abotalebmostafa@yandex.ru

Nowadays much attention has been paid to the development of the software, which makes it possible to process and visualize images, and in particular, develop the mathematical models for monitoring and predicting of crops quality to optimize the profit. Modelling the dynamics of vegetation indices in this regard is very relevant and important task especially if it is made with high level of detailing. A number of research is underway in the agricultural sector to better predict crop yield using machine learning algorithms. In our research we suggest the deterministic approach using general least deviation method to obtain the model for a dynamic process. Using the obtained models data from the previous periods we can compare them with data for the current one and forecast the development of the crops in the current vegetation period. Unlike neural networks, this approach makes it possible to explicitly obtain high-quality quasi-linear difference equations for any field and any region. We use this model for modelling of normalized difference vegetation index dynamics for several years and discuss the opportunities to use our algorithm as a base of software for crop prediction, and detection the problem areas of a field.

keywords: forecasting, time series, quasilinear model, generalized least deviations method, monitoring crop yields, NDVI dynamics.

Vsevolod Voronov[1,2], **Alexander Tolmachev**[2,3], **Dmitry Protasov**[2,4] **and Anna Neopryatnaya**[2,3]

SEARCHING FOR DISTANCE GRAPH EMBEDDINGS AND OPTIMAL PARTITIONS OF COMPACT SETS IN EUCLIDEAN SPACE

[1] Caucasus Mathematical Center of Adyghe State University, Russia
[2] Moscow Institute of Physics and Technology, Russia
[3] Skolkovo Institute of Science and Technology, Russia
[4] Artificial Intelligence Research Institute, Russia
v-vor@yandex.ru

We consider three problems in combinatorial geometry in which the search for counterexamples and improvement of known estimates is reduced to a finite-dimensional multi-extremal optimization problem with piecewise-smooth constraints. The first problem is to find a distance embedding of some graph into a given surface, i.e. to find a set of points for which a part of pairwise distances and some additional condition are given. The other two problems consist in minimization of some functional computed for partitions of a compact set into a given number of subsets. The solutions found have improved some quantita-

tive estimates in generalizations of the Borsuk hypothesis and variants of the Hadwiger–Nelson–Erdös problem on the chromatic number of space.

keywords: Borsuk problem, distance graphs, stochastic gradient descent, global optimization.

Mostafa Abotaleb[1], **Amr Badr**[2], **Tatiana Makarovskikh**[1] **and Anatoly Panyukov**[1]

Grid search to optimize LSTM hyperparameters for daily infections SARS-CoV-2 cases in Russia

[1] South Ural State University, Russia
[2] University of New England, Australia
makarovskikh.t.a@susu.ru, paniukovav@susu.ru
abotalebm@susu.ru, amr.mostafa@live.com

The most popular technique in deep learning to handle univariate time series is the long short-term memory (LSTM) model, which is used to recognize the pattern of problems that have non-linear behavior. In our work, we developed an algorithm for optimizing a long short-term memory (LSTM) model by automatically tuning the hyperparameter, and we have applied it to daily SARS-CoV-2 infection cases in Russia, which is distinguished as a nonlinear dataset of high complexity. By using the grid search method, which basically works by defining a subset of candidate values for each hyperparameter and training all the possible combinations of the hyperparameters, Then, each possible fitted model is evaluated on a validation set, and the best configuration of the hyperparameter will be chosen at the end. The achieved results confirm the effectiveness, superiority, and significance of the proposed approach in predicting the infection cases of SARS-CoV-2, where approximately an improvement in RMSE of 30% was observed.

keywords: LSTM, optimizing, grid search, deep learning, SARS-CoV-2.

Tatiana Antonova[1] **and Aleksandr Ageev**[1]

A new approach to the localizing fractal lines of discontinuity

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
ageev@imm.uran.ru, tvantonova@imm.uran.ru

The ill-posed problem of localizing (finding the position of) discontinuity lines from noisy data arises in image processing, when the boundaries of image objects are lines on which a function of two variables (image) suffers a discontinuity of the first kind (discontinuity lines); outside the discontinuity lines, the function is smooth. The boundaries of natural objects often cannot be described by smooth lines therefore line classes including fractal curves, are introduced; regularizing algorithms for localizing discontinuity lines on these classes are constructed and substantiated.

keywords: ill-posed problem, regularizing algorithms.

Artyom Firstkov, Majid Forghani and Michael Khachay

Searching optimal embedding for genetic sequences with application in modeling of viral evolution

[1] Krasovsky Institute of Mathematics and Mechanics, Russia
firstk121@gmail.com, mkhachay@imm.uran.ru

Embedding of finite metric configurations to finite-dimensional Euclidean spaces is the well-known preliminary solution stage for many relevant problems in bioinformatics. Recently, application of embedding techniques originated from Natural Language Processing (NLP), e.g. word2vec framework, became more applicable for representing the genetic sequences. To select an adequate embedding, it is important to assess how effectively it reflects biological similarities of the sequences. Such a characteristic could be the phylogenetic distance between considered strains of the virus. To estimate how embedding preserves the phylogenetic distances between strains, we use distortion metric and the SNE loss function. By carrying out computational experiments, we give a numerical efficiency proof of the selected embedding for mathematical modeling of the antigenic evolution of the influenza virus. We beleive that the proposed approach can be extended to more wide class of phenotype modelling problems on the basis of genetic sequences.

keywords: embedding, word2vec, modeling antigenic evolution, optimization, genetic sequence.

Petr Koldanov[1], Aleksander Koldanov[1] and Dmitriy Semenov[1]

Investigation of uncertainty usign upper and low confidence bounds

[1] National Research University Higher School of Economics,Russia
pkoldanov@hse.ru, akoldanov@hse.ru, dpsemenov@hse.ru

Random variable network is a general model related with biological and medical studies, gene expession or gene co-expression analysis, market network analysis, climate network analysis and others.Different graph structures are used to emphasize some important information in network. Simple and popular graph structure in random variable network is a threshold similarity graph. Threshold similarity graph emphasize a strengths and topology of connections in the network, and it is known as market graph in market network analysis. One of the most important problem related with graph structures is uncertainty of it's identification by observations.To handle the uncertainty we propose to construct an upper and low confidence bounds for threshold similarity graph. These bounds allows to make significant conclusions on edges inclusions in threshold similarity graph. Note that an interest to the methods of statistically significant network analysis has been increased last decades. Stability of these confidence bounds is investigated as well. A method to construct upper and low confidence bounds for the threshold similarity graph identification is

proposed. The method is based on application of multiple hypotheses testing procedures with FWER control in strong sense. Obtained confidence bounds allow to derive statistically significant conclusions on edges for the threshold similarity graph. In particular, it is possible to identify the statistically significant set of edges included in the threshold graph, and simultaneously the statistically significant set of edges not included in the threshold graph. To investigate stability of confidence probability with respect to distribution three random variable networks are considered in the wide class of elliptical distributions. Obtained results shows that for the case of normal distribution of the vector X Bonferroni type procedure to upper and low confidence bounds construction based on Pearson correlation tests is stable. From the other side this not true for the case of deviation from normal distribution. Namely, it is shown that Bonferroni type procedure based on Pearson correlation tests does not control the confidence probability for the case of elliptical distributions with heavy tails. The interesting results concerning Kendall correlation network are obtained. Namely despite from the weak instability of Kendall correlation test in the class of elliptically contoured distributions the Bonferroni type procedure to upper and low confidence bounds construction in Kendall correlation network is stable. These experimental results allow to recommend the Bonferroni type procedure based on Kendall correlation tests for practical applications.

keywords: random variable network, uncertainty of graph structure identification, upper and low confidence bounds, multiple hypotheses testing.

Nikolay Kutuzov[1], **Alexander Gasnikov**[1,2,3,4], **Eduard Gorbunov**[5], **Yuriy Dorn**[1] **and Alexander Nazin**[6]

Implicitly normalized forecaster with clipping for heavy-tailed MAB problem

[1] Moscow Institute of Physics and Technology, Russia
[2] Vernadsky Crimean Federal University, Russia
[3] HSE University, Russia
[4] Kharkevich Institute for Information Transmission Problems, Russia
[5] Caucasus Mathematical Center, Adyghe State University, Russia
[6] Mohamed bin Zayed University of Artificial Intelligence, UAE
[7] Trapeznikov Institute of Control Sciences, Russia
nazin.alexander@gmail.com

Implicitly Normalized Forecaster (online mirror descent with Tsallis entropy as prox) is known to be an optimal algorithm for adversarial multi-armed problems. However, most of the complexity results assumed bounded rewards or other similar limitations. One of the important problem classes that do not satisfy standard assumptions is adversarial MAB with heavy tailed distribution on rewards. In this paper we propose Implicitly Normalized Forecaster with

clipping for multi-armed problems with heavy-tailed distribution on rewards. We derive convergence results under mild assumptions on rewards distribution and show that proposed method is optimal.

keywords: multi-armed bandits, clipping, online mirror descent.

Igor Masich[1]

Optimization of the support set of attributes and formal concepts analysis for the conceptual clustering of elements of the electronic component base.

[1] Reshetnev Siberian State Aerospace University, Russia

A number of real problems of automatic grouping of objects or clustering require a reasonable solution and the possibility of interpreting the result. One of these problems is the problem of identifying homogeneous subgroups of elements of the electronic component base. The number of groups in such a data set is not specified, and it is required to justify and describe the proposed grouping model. As a tool for interpretable machine learning, we consider formal concept analysis (FCA). To reduce the problem with real attributes to a problem that allows the use of AFP, we use the search for the optimal number and location of cut points and the optimization of the support set of attributes. Revealed concepts are evaluated by indicators of informativeness and can be considered as homogeneous subgroups of elements and their indicative description. The proposed approach makes it possible to single out homogeneous subgroups of elements and provides a description of their characteristics, which can be considered as tougher norms that the elements of the subgroup satisfy.

keywords: automatic grouping of objects, conceptual clustering, machine learning, formal concept analysis, homogeneous subgroups of elements, indicative description.

Aleksandr Maslovskiy[1] **and Alexander Degtyarev**

Attention technique for self-interference cancellation in full-duplex systems

[1] Moscow Institute of Physics and Technology, Russia

This article investigates solution to the non-linear self-interference problem in full-duplex (FD) wireless communication systems. Traditionally, self-cancellers are made up of linear and non-linear blocks, with non-linear blocks based on polynomials. Nonetheless, such models are hampered by their tremendous complexity. There is an alternate proposal, an active self-interference cancellation

approach for full-duplex systems based on the recurrent neural network (RNN) with attention technique, which has been well advised in digital-predistortion scenarios. This strategy enables us to attain high performance in the present work for a variety of instances.

keywords: full-duplex (FD), RNN, IGRNN, TPA.

Daria Semenova[1], **Aleksandr Soldatenko**[1] **and Ellada Ibragimova**[1]

On heuristic algorithm with greedy strategy for the Correlation Clustering problem solution

[1] Siberian Federal University, Krasnoyarsk, Russian Federation
DVSemenova@sfu-kras.ru, ASoldatenko@sfu-kras.ru, IbragimovaEI@mail.ru

The Correlation Clustering (CC) problem traditionally defined as a problem of splitting a signed graph without specifying the number of clusters is considered. In practice this problem arises in the analysis of social networks and multi-agent systems. In this paper, CC problem is solved for undirected and unweighted signed graphs without multiple edges and loops, where error functional is linear combination of intercluster and intracluster errors. In this formulation, the CC problem is NP-complete. Exact algorithms for this problem are time-consuming. Approximate algorithms for solving CC problem often lead to unsatisfactory results, and heuristic algorithms are often non-deterministic in the number of steps leading to a solution. We propose a new heuristic algorithm $SGClust_\alpha$ for CC problem solving. The main idea of this algorithm is in minimizing of intracluster error and optimization of error functional according to the greedy strategy. It was proved that this algorithm takes polynomial time. Numerical experiments were carried out on randomly generated signed graphs and real dataset presented in the literature and open databases.

keywords: correlation clustering, signed graph, heuristic, greedy algorithm, structural balance, graph partition.

Qiushi Sun[1], **Yuyi Zhang**[1], **Haitao Wu**[1] **and Ovanes Petrosian**[1]

Deep neural network based resource allocation in D2D wireless networks

[1] Saint Petersburg State University, Russia
st059656@student.spbu.ru, st088518@student.spbu.ru
st082606@student.spbu.ru, o.petrosyan@spbu.ru

The increased complexity of future 5G wireless communication networks presents a fundamental issue for optimal resource allocation. A key challenge is the

power control to the individual antennas. This is a nondeterministic polynomial (NP) problem that needs to be solved in time since the power allocation should be consistent with the instantly evolving channel state. This paper emphasizes the application of Deep learning (DL) to develop solutions for radio resource allocation problems in multiple-input multiple-output (MIMO) systems. We introduce a supervised Deep neural network (DNN) model combined with particle swarm optimization (PSO) to address the issue using heuristic-generated data. We train the model and evaluate its ability to anticipate resource allocation solutions accurately. The simulation result indicates that the trained DNN-based model can deliver the desired optimal solution.

keywords: MIMO, deep neural networks, heuristics, PSO.

Grigoriy Tamasyan[1,2] **and George Shulga**[3]

SUM OF MODULES OF AFFINE FUNCTIONS OPTIMISATION: LINEAR REGRESSION, METHODS AND COMPARISON

[1] Mozhaiskiy Space Military Academy, Russia
[2] Institute of Problems of Mechanical Engineering, Russia
[3] Saint Petersburg State University, Russia
grigoriytamasjan@mail.ru, gdextrous@gmail.com

It is generally accepted to consider linear regression in the standard Euclidean metric, but in practice it turns out to be useful also considering linear regression in the octahedral (L1) metric. In cases when no information about measurement errors is given, this is equivalent to minimizing the sum of modules of several affine functions. Methods for solving such a problem (one-dimensional and multidimensional) will be discussed, a comparative analysis will be carried out, and some new approaches will be suggested. In particular, in the multidimensional case, we propose a new effective method, which was obtained using nondifferentiable optimization and the theory of linear programming.

keywords: linear regression, sum of modules, affine functions, octahedral metric, nonsmooth analysis, linear programming.

Sergey Vandanov[1], **Aleksandr Plyasunov**[2] **and Anton Ushakov**[1]

DEVELOPMENT OF A PARALLEL CLUSTERING ALGORITHM FOR THE K-MEDOIDS PROBLEM IN HIGH-DIMENSIONAL SPACE FOR LARGE-SCALE DATASETS

[1] Novosibirsk State University, Russia
[2] Matrosov Institute for System Dynamics and Control Theory, Russia
apljas@math.nsc.ru

The k-medoids (k-median) clustering problem is an important problem in the field of unsupervised learning. It is used to partition a set of data points into k clusters, where each cluster is represented by one of the data points, called

a medoid. This problem is widely used in various fields such as data mining, machine learning, and image processing. It has been applied to a variety of real-world applications such as data reduction, anomaly detection, and market segmentation. K-medoids algorithm is more robust to noise and has an ability to handle different types of data, including categorical and numerical data.

Despite its importance, finding high-quality solutions for large-scale datasets remains a challenging task. In this paper, we improve previous work for the k-medoids problem [1] and propose a parallel primal-dual heuristic algorithm in high-dimensional space. We developed an algorithm that addresses the limitations of existing solutions. These limitations include: time-consuming calculation of the distance matrix, unefficient searching for nearest neighbours, the absence of efficient GPU parallel algorithms, difficulties working with large-scale datasets.

Our algorithm employs an efficient parallel implementation of nearest neighbor search, *a parallel subgradient column generation method, and a novel subgradient search algorithm.* During the work, a comprehensive literature review was conducted to critically evaluate contemporary methods for addressing k-means, nearest neighbour, and k-medoids problems using both CPU and GPU-based approaches. The review culminated in the identification of the most promising algorithms, which were subsequently chosen for in-depth study with the aim of incorporating their key principles into the design and implementation of a novel algorithm.

As a result of the research, the execution time of the algorithm on the BIRCH dataset was studied in relation to variations in the initial parameters and different implementations of specific components of the algorithm. Additionally, a comparative analysis of nearest neighbor search times was performed, utilising algorithm [2] as a reference point. Furthermore, the feasibility of applying the techniques proposed in article [3] to compute the approximate distance matrix was explored.

Through our experiments, we demonstrate that our algorithm finds high-quality solutions for large-scale datasets and outperforms this k-medoids clustering algorithms by achieving significant speedup.

1) A. Ushakov, I. Vasilyev. Near-optimal large-scale k-medoids clustering, Information Sciences., 2020

2) V. Garcia. Fast k-nearest neighbor search using GPU, 2009

3) J. Johnson, M. Douze. Billion-scale similarity search with GPUs, 2017

keywords: parallel clustering algorithm, the k-medoids problem, sub-gradient optimization.

Anastasia Zhadan[1], **Haitao Wu**[1], **Pavel Kudin**[1], **Yuyi Zhang**[1] **and Ovanes Petrosian**[1]

MICROGRID CONTROL FOR RENEWABLE ENERGY SOURCES BASED ON DEEP REINFORCEMENT LEARNING AND NUMERICAL OPTIMIZATION APPROACHES

[1] Saint Petersburg State University, Russia
a89173627785@gmail.com, accpavel1@gmail.com,
lesliezhang0825@gmail.com, petrosian.ovanes@yandex.ru

Optimal scheduling of battery energy storage system plays crucial part in distributed energy system. As a data driven method, deep reinforcement learning does not require system knowledge of dynamic system, present optimal solution for nonlinear optimization problem. In this research, financial cost of energy consumption reduced by scheduling battery energy using deep reinforcement learning method (RL). Reinforcement learning can adapt to equipment parameter changes and noise in the data, while mixed-integer linear programming (MILP) requires high accuracy in forecasting power generation and demand, accurate equipment parameters to achieve good performance, and high computational cost for large-scale industrial applications. Based on this, it can be assumed that deep RL based solution is capable of outperform classic deterministic optimization model MILP. This study compares four state-of-the-art RL algorithms for the battery power plant control problem: PPO, A2C, SAC, TD3. According to the simulation results, TD3 shows the best results, outperforming MILP by 5% in cost savings, and the time to solve the problem is reduced by about a factor of three.

keywords: Reinforcement Learning, Energy Management System, Distributed Energy System, Numerical Optimization.

Author Index

Ablaev S., 19
Abotaleb M., 59, 109, 110
Ageev A., 110
Ahmatshin F., 60
Aida-Zade K., 5
Aivazian G., 19
Akmaeva V., 46
Alekseev A., 97
Alikhani S., 34
Alkousa M., 17, 19
Ananyev B., 92
Anikin A., 45, 63
Antipin A., 101
Antonova T., 110
Antsyz S., 99
Arkhipov A., 46
Arkhipov D., 60
Aroslankin A., 108

Badr A., 59, 110
Baran I., 19
Barkova M., 20
Battaïa O., 38, 64
Berdin E., 52
Berenov D., 74
Beresnev V., 71, 88
Berikov V., 108
Bobrov E., 56, 69, 77
Borisovsky P., 61
Bulavchuk A., 35
Burashnikov E., 35
Bykadorov I., 100

Chentsov A., 56
Chentsov P., 56
Cherikbayeva L., 108
Chernykh I., 50
Chezhegov S., 15
Chirkov A., 35
Chirkova J., 80
Chistyakov V., 100
Chukanoc S., 45

Danik Y., 92
Davydov I., 28, 64
Degtyarev A., 113
Dmitriev M., 92, 102
Dobrynin A., 36
Dordzhiev A., 56
Dorn Y., 112

Enkhbat R., 21, 83
Eremeev A., 36, 62
Erokhin V., 20
Ershov A., 97
Ershov M., 44, 47
Ershova A., 97
Erzin A., 28, 29

Fedin N., 15
Firstkov A., 111
Fominyh A., 93
Forghani M., 111

Gao H., 89
Garbar S., 44
Gasnikov A., 15, 17, 25, 26, 45, 47, 70, 112

Gladyshev D., 62
Gladyshev S., 62
Glotova Y., 37
Gnusarev A., 57
Golmohammadi H., 34
Golovanov S., 63
Gomoyunov M., 80
Gompil B., 21
Gorbunov E., 15, 112
Gornov A., 45, 63, 83
Gorynya E., 52
Gribanov D., 29, 35
Gruzdeva T., 16
Guarracino M., 5
Gubar E., 67
Guliev S., 5
Gusev M., 93
Gusev V., 84
Guseva S., 94

Ianovski E., 67
Ibragimov D., 94
Ibragimova E., 114
Ignatov A., 44
Il'ev V., 30
Il'eva S., 30
Ivanov S., 46
Ivashko A., 81
Ivashko E., 72
Izmest'ev I., 81

Jaćimović M., 6

Kalmutskiy K., 108
Kalyagin V., 108
Kareeva Yu., 84
Kazakov A., 69
Kazakovtsev L., 22, 23, 60, 63, 75
Kazakovtsev V., 63
Khachai D., 38, 64, 76
Khachay M., 38, 40, 64, 76, 111
Khamisov O., 21, 22, 65
Khandeev V., 31
Kharchenko Ya., 38
Khmara I., 66
Khoroshilova E., 101
Kibzun A., 46
Kochetov Y., 41, 70
Kochetov Yu., 58, 78
Kochevadov V., 66
Koldanov A., 111
Koldanov P., 111
Kolnogorov A., 47
Kolosov D., 77
Kondratev A., 67
Kononov A., 29, 38
Kononova P., 39
Konstantinova E., 34
Korostil A., 31
Kosyanov N., 67
Krasilnikov M., 13
Krivonogova O., 50
Krivorotko O., 87, 104
Krutikov V., 22, 23
Krylatov A., 68
Kudin P., 117
Kudryavtsev K., 85
Kulachanko I., 39
Kulachenko I., 58
Kumacheva S., 101
Kumkov S., 72, 77
Kuruzov I., 24
Kutuzov N., 112
Kuzenkov O., 24
Kuzyutin D., 85

Ladygin I., 28
Lavlinskii S., 56, 78
Lazarev A., 51
Lebedev P., 69, 94
Lempert A., 69
Levanova T., 57, 66
Litvinenko A., 108
Lobanov A., 45, 47
Logachev A., 52
Loktev S., 69

Lozkins A., 52
Lu P., 6
Lubnina A., 70
Lyapin A., 64

Makarov D., 102
Makarovskikh T., 59, 109, 110
Malakh S., 52
Marakulin V., 95
Masich I., 113
Maslovskiy A., 70, 113
Mazalov V., 81
Mazurov V., 71
Melnikov A., 58, 71, 88
Mikhailov A., 72
Muftahov I., 51

Nazarenko S., 29
Nazin A., 112
Neopryatnaya A., 109
Neshchadim S., 31
Nesterov A., 67
Neverov A., 87
Neznakhina K., 40
Nikitina N., 72
Nikolaev A., 31
Novikov D., 103
Novitskii A., 25

Obrosova N., 9
Ogorodnikov Y., 76
Orlov A., 87, 88
Osipov I., 93

Panin A., 56, 73
Pankratova Y., 82, 85
Panyukov A., 59, 109, 110
Pardalos P., 6
Parilina E., 82
Perov D., 24
Petrosian O., 89, 114, 117
Petrosyan L., 82, 83
Petrova I., 60
Petunin A., 64
Pinyagina O., 16
Plotnikov R., 28
Plyasunov A., 56, 73, 115
Posypkin M., 21, 40
Prokudina L., 73
Protasov D., 109
Pyatkin A., 32

Raevskaya A., 68
Rasskazova V., 74
Ren J., 60
Rettieva A., 81
Rezova N., 75
Ripatti A., 33
Rizhenko K., 40
Rogozin A., 15, 25, 26
Rozenberg V., 95
Rozhnov I., 75
Rubtsova E., 57
Rudakov R., 76

Sadykov R., 38
Sakhno M., 54
Samsonyuk O., 104
Savchuk O., 17
Sedakov A., 66, 84
Seliverstov A., 40
Semenikhin K., 46
Semenkin E., 7
Semenov D., 111
Semenova D., 35
Semenova D., 114
Sergeyev Ya., 8
Servakh V., 52
Sevastyanov S., 53
Sevostyanov R., 96
Shananin A., 9, 96
Sharankhaev K., 29
Sharf A., 71
Shkaberina G., 63, 75
Shmyrina A., 50
Shperling S., 41

Shulga G., 115
Shulgina O., 18
Sigaev V., 57
Silaev D., 62
Simanchev R., 33
Skachkov D., 60
Smirnova N., 85
Sofronova E., 77
Soldatenko A., 114
Sorokovikov P., 63, 83
Sotnikova M., 96
Spiridonov A., 77
Stanimirović P., 10
Stanimirovic P., 22, 23
Stonyakin F., 17, 19, 24
Strekalovsky A., 20, 25
Su Sh., 82
Sukhee B., 21
Sumenkov O., 70
Sun Q., 114

Tamasyan G., 115
Tarabukin I., 70
Tarasyev A., 98
Taynitskiy V., 67
Tikhomirov A., 48
Titov A., 17
Tolmachev A., 109
Tomilina G., 101
Tovbis E., 22, 23
Trusov N., 96
Tur A., 83

Ukhobotov V., 81
Ulyanova N., 65
Urazova I., 33
Ushakov A., 16, 28, 51, 115
Ushakov V., 97
Usova A., 98
Uspenskii A., 94
Ustyugov V., 36
Utyupin S., 88

Vandanov S., 115
Vasilyev I., 28, 51
Vasin V., 11
Veselov S., 35
Vikhirev M., 73
Vodyan M., 73
Vorkutov D., 70
Vorobeva I., 104
Voronov V., 109
Voroshilov A., 44, 47

Wang Z., 89
Wu H., 114, 117

Yarmoshik D., 26
Yarullin R., 17, 18
Yeung D., 82
Yudakov D., 77
Yudin N., 26
Yue Z., 69
Yuskov A., 58

Zabirova R., 17
Zabotin I., 17, 18
Zabudsky G., 34
Zakharov A., 41
Zakharova Y., 50, 54
Zaozerskaya L., 36, 42
Zarodnyuk T., 63, 83
Zhadan A., 117
Zhang D., 60
Zhang Y., 114, 117
Zhen M., 84
Zhitkova E., 101
Zhou J., 89
Zhukov I., 90
Zolotykh N., 35
Zubov V., 18
Zvonareva T., 104
Zykov I., 105
Zyryanov A., 78

XXII INTERNATIONAL CONFERENCE MATHEMATICAL OPTIMIZATION THEORY AND OPERATIONS RESEARCH (MOTOR 2023)

Abstracts

Michael Khachay, et al (Eds.)

SCIENTIFIC EDITION

TEX-редактор К. В. Рыженко

Подписано к печати .06.23. Формат $60 \times 84^1/_{16}$.
Печать офсетная. Усл. печ. л. , . Уч.-изд. л. , .
Тираж 200 экз. Заказ

Федеральное государственное бюджетное учреждение науки
Институт математики и механики им. Н. Н. Красовского
Уральского отделения Российской академии наук
620108, г. Екатеринбург, ул. С. Ковалевской, 16.

Размножено с готового оригинал-макета в типографии
ООО "Издательство УМЦ УПИ"
620078, г. Екатеринбург, ул. Гагарина, 35а, оф. 2.
тел. (343)362-91-16, 362-91-17